STANDARD ON
Ratio Studies

A criterion for measuring fairness, quality, equity and accuracy

(Approved April 2013)

IAAO

INTERNATIONAL ASSOCIATION
of ASSESSING OFFICERS

Standard on Ratio Studies

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Standard on Ratio Studies

Part 1. Guidance for Local Jurisdictions

This standard comprises two major parts. Part 1 focuses on the needs of local assessors. Part 2 presents guidelines for oversight agencies that use ratio studies for equalization and appraisal performance monitoring. The Definitions section explains the terms used in this standard. The appendixes present many technical issues in greater detail. More information on many topics addressed in this standard can be found in Property Appraisal and Assessment Administration (IAAO 1990, chapter 20) and in Gloudemans (1999, chapter 5).

1. Scope

This part of the standard provides recommendations on the design, preparation, interpretation, and use of ratio studies for the real property quality assurance operations of an assessor's office. Quality assurance/control measures include data integrity review, assessment level and uniformity analysis, and computer-assisted mass appraisal (CAMA) system performance testing, among others.

Assessors may have the opportunity to utilize ratio study information at a greater depth than oversight agencies. These internal studies can help improve appraisal methods or identify areas within the jurisdiction that need attention. External ratio studies conducted by oversight agencies (Part 2) focus more upon testing the assessor's past performance in a few broad property categories.

2. Overview

For local jurisdictions, *ratio study* is used as a generic term for sales-based studies designed to evaluate appraisal performance. The term is used in preference to the term *assessment ratio study* because use of assessments can mask the true level of appraisal and confuse the measurement of appraisal uniformity when the legal assessment level is other than 100 percent of fair market value.

2.1 The Concepts of Market Value and Appraisal Accuracy

Market value is the major focus of most mass appraisal assignments. The major responsibility of assessing officers is estimating the market value of properties based on legal requirements or accepted appraisal definitions. The viability of the property tax depends largely on the accuracy of such value estimates. The accuracy of appraisals made for assessment purposes is therefore of concern, not only to assessors but also to taxing authorities, property taxpayers, and elected representatives. Appraisal accuracy refers to the degree to which properties are appraised at market value, as defined by professional standards (see *Glossary for Property Appraisal and Assessment* [IAAO 1997]) and legal requirements. While a single sale may provide an indication of the market value of the property in question, it cannot form the basis for a ratio study, which provides information about the market values of groups of properties. Dividing the appraised value by the sale price forms the ratios. The ratio can be multiplied by 100 and expressed as a percentage.

Market value is a concept in economic theory and cannot be observed directly. However, market values can be represented in ratio studies by sales prices (market prices) that have been confirmed, screened, and adjusted as necessary (see Appendix A, "Sales Validation Guidelines"). Sales prices provide the most objective estimates of market values and under normal circumstances should provide good indicators of market value.

2.2 Aspects of Appraisal Performance

There are two major aspects of appraisal accuracy: level and uniformity. Appraisal level refers to the overall ratio of appraised values to market values. Level measurements provide information about the degree to which goals or certain legal requirements are met. Uniformity refers to the degree to which properties are appraised at equal percentages of market value.

2.3 Uses of Ratio Studies

Key uses of ratio studies are as follows:

- measurement and evaluation of the level and uniformity of mass appraisal models

- internal quality assurance and identification of appraisal priorities

- determination of whether administrative or statutory standards have been met

- determination of time trends

- adjustment of appraised values between reappraisals

Assessors, appeal boards, taxpayers, and taxing authorities can use ratio studies to evaluate the fairness of funding distribution*s*, the merits of class action claims, or the degree of discrimination (see Appendix G). However, ratio study statistics cannot be used to judge the level of appraisal of an *individual* parcel. Such statistics can be used to adjust assessed values on appealed properties to the common level.

2.4 Applicability

Local jurisdictions should use ratio studies as a primary mass appraisal testing procedure and their most important performance analysis tool. The ratio study can assist such jurisdictions in providing fair and equitable assessment of all property. Ratio studies provide a means for testing and evaluating mass appraisal valuation models to ensure that value estimates meet attainable standards of accuracy; see *Uniform Standards of Professional Appraisal Practice* (USPAP) *Standard Rule 6-6* (Appraisal Foundation 2010-2011). Ratio study reports are typically included as part of the written documentation used to communicate results of a mass appraisal and to comply with *Standard Rule 6-7(b)*. Ratio studies also play an important role in judging whether constitutional uniformity requirements are met. Compliance with state or provincial performance standards should be verified by the local jurisdiction before value notices are sent to property owners.

3. Steps in Ratio Studies

Ratio studies generally involve the seven basic steps listed below.

1. define the purpose, scope and objectives

2. design

3. stratification

4. collection and preparation of market data

5. matching of appraisal and market data

6. statistical analysis

7. evaluation and use of results

3.1 Definition of the Purpose, Scope, and Objectives

The first step in any ratio study is to determine and state clearly the reasons for the study. This crucial step of identifying the purpose of the study determines the specific goals, scope, content, depth, and required flexibility.

3.2 Design

In the design of the study the assessor must consider the quantity of sale data and the resources available for conducting the ratio study. Although absolute accuracy cannot be ensured, all reasonable, cost-effective steps should be taken to maximize reliability.

The assessor should identify the following factors:

- the groups or classes of properties to be included in the study

- important legal, physical, and economic characteristics of the properties selected for study

- the quantity and quality of data available

- the values being tested and sales period being used

- available resources, such as the number and expertise of staff, computer hardware and software applications, and additional limiting conditions

3.2.1 Level of Sophistication and Detail

A basic design principle is to keep the study as simple as possible while consistent with its purpose. Ratio studies are not all alike and should be tailored to an intended use.

Data analysis has been made easier through computerization. Although every study does not require the same level of statistical detail, each ratio study should include measures of appraisal level, appraisal uniformity, and statistical reliability. Graphs, charts, or other pictorial representations can be useful tools for showing distributions and patterns in the data. There is no model ratio study design that can serve all jurisdictions or all situations equally well. Informed, reasoned judgment and common sense are required in the design of ratio studies.

3.2.2 Sampling

A ratio study is a form of applied statistics, because the analyst draws conclusions about the appraisal of the population (the entire jurisdiction) of properties based only on those that have sold during a given time period. The sales ratios constitute the sample that will be used to draw conclusions or inferences about the population.

To determine the accuracy of appraisals with absolute certainty, it would be necessary for all properties in the population to have been sold in arm's-length, open-market transfers near the appraisal date. Since this is not possible, ratio studies must use samples and draw inferences or conclusions about the population from these samples.

The number of parcels in the population (the jurisdiction or stratum) is not an important determinant of a statistically valid and reliable sample.

3.2.2.1 Limitations of Sale Samples

Users of sales ratio studies should be aware of the following cautions associated with use of sale samples:

- Depending on the circumstances, sales prices can provide either useful or poor indications of market values. Sales must be screened to eliminate those that don't meet the requirements of arm's-length, open-market sales (see *Standard on Verification and Adjustment of Sales* [IAAO 2010]).

- Sales are not "randomly selected" from the population, in the strict technical sense (see section 4.5, Sample Representativeness).

- Value-related characteristics of a sale sample may not represent all the value-related characteristics of the population.

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- Adjustments to sale prices may be difficult to support or may be subjective.

3.2.2.2 Data Accuracy and Integrity

The findings of a ratio study can only be as accurate as the data used in the study. Personnel involved in collecting, screening, and adjusting sales data or making appraisals should be familiar with real estate conveyance practices in their region. They also should be proficient in the principles and practices of real estate appraisal and understand local market conditions.

Accuracy and integrity of data entered into or transferred through computer systems must be ensured. Design of computer programs should make it easy to verify data accuracy. Query tools should be accessible to users, so that data can be verified easily. Methods for checking the accuracy of assigned strata (such as school district, city, neighborhood, and category) as well as of assessed or appraised value, sale price, parcel identifier, and other fields must be established to reduce these and other nonsampling errors.

3.3 Stratification

Stratification divides all the properties within the scope of the study into two or more groups or strata. Stratification facilitates a more complete and detailed picture of appraisal performance and can enhance sample representativeness.

Each type of property subject to a distinct level of assessment could constitute a stratum. Other property groups, such as neighborhoods and age and size ranges, could constitute additional strata.

When the purpose of the study is to evaluate appraisal quality, flexibility in stratification is essential. The general goal is to identify areas in which the assessment levels are too low or lack uniformity and property groups for which additional reappraisal work may be required. In such cases, it also is highly desirable to stratify on the basis of more than one characteristic simultaneously.

Stratification can help identify differences in level of appraisal between property groups. In large jurisdictions, stratification by geographic areas is generally more appropriate for residential properties, while stratification of commercial properties by either geographic area or property subtypes (e.g., office, retail, and warehouse/industrial) can be more effective.

3.4 Collection and Preparation of Market Data

The reliability of a ratio study depends in part on how well the sales used in the study reflect market values. The underlying principle for review of sales data is to optimize the sample size, but at the same time to exclude sales that provide invalid indicators of market value. A ratio study

sample with fewer than five sales tends to have exceptionally poor reliability and is not very useful.

3.5 Matching of Appraisal and Market Data

The physical and legal characteristics of each property used in the ratio study must be the same as when sold. This implies two essential steps. First, the appraiser must ascertain whether the property descriptions match. If a parcel is split between the appraisal date and the sale date, a sale of any of its parts should not be used in the ratio study.

Second, the appraiser must ascertain whether the property rights transferred, the permitted use, and the physical characteristics of the property on the date of assessment are the same as those on the date of sale. If the physical characteristics of the property have changed since the last appraisal, adjustments may be necessary before including the property in a ratio study. Properties with significant differences in these factors should be excluded from the ratio study.

When statutory constraints are imposed on appraisal methods, the resulting assessment may be less than market value. In such cases a sales ratio study may not provide useful performance information. Constraints typically apply to land that qualifies for agricultural use value, subsidized housing, mineral land, and timberland.

Sales may include property of a type other than the type for which the ratio study analysis is intended. However, sales including more than minimal values of secondary categories are unlikely to be representative, even with adjustment.

For example, a property that is predominantly commercial may include residential components. This sale can be included as representative of the commercial category. In this case, the numerator in the ratio calculation would be the total appraised value including the value of both the commercial and residential components.

In a second example, for a ratio study of vacant land, the numerator in the ratio should reflect only the appraised value of the land. The sale price should be adjusted for the contributory value of the improvements or the sample should be excluded from further analysis.

3.6 Statistical Analysis

After sales have been screened and matched against assessed values, ratios computed, and outliers identified and removed if appropriate, measures of appraisal level, uniformity, and reliability for the entire jurisdiction and each group or stratum should be computed. The sample also could undergo exploratory data analysis to reveal patterns or features of the data (Hoaglin, Mosteller, and Tukey 1983).

3.7 Evaluation and Use of Results

A properly designed ratio study is a powerful tool for analyzing appraisal performance, evaluating CAMA system models, and suggesting strategies for improvement. A ratio study also can identify weaknesses in appraisal system performance. Unexpected study results may indicate a need to respecify or recalibrate an appraisal model or to reevaluate the data elements used in the valuation process. However, users of ratio studies should recognize the inherent limitations of this tool, as follows:

1. A ratio study cannot provide perfect information about appraisal performance. Lack of sufficient sales, outliers, or overrepresentation of one geographic area or type of property can distort results.

2. Ratio study validity requires that sold and unsold parcels be appraised at the same level and in the same manner. Violation of this condition seriously undermines the validity of the study.

3. Findings should be used only in ways that are consistent with the intended use(s) for which the study was designed.

4. Ratio study data are subject to statistical sampling errors and other processing (nonsampling) errors (see Lessler and Kalsbeek), but these limitations do not invalidate their use for informed decision-making.

4. Timing and Sample Selection

4.1 Data Requirements and Availability

The availability of data influences the design of the study and can call for revisions in the objectives of the study, limit the usefulness of the calculated statistics, or both.

4.1.1 Nature of the Population

The type of properties, market conditions, and composition of the population in terms of age, size, and value range are essential to the proper design of the study and interpretation of the results. Very large properties that rarely sell (e.g., a large power plant) can be ignored in a ratio study designed to evaluate local appraisal performance.

4.1.2 Assessment Information

Appraised values are the numerators in the ratios used in a ratio study. Information about appraisal dates, legal requirements concerning reappraisals, the dates on which the appraisals were originally set, and the period they remained in effect is required for establishing the date of analysis.

4.1.3 Indicators of Market Value

Sale price, as an indicator of market value, is the denominator in the calculation of the ratio. Specific information about the date, amount, terms, and conditions of a sale is required for proper analysis.

4.1.4 Property Characteristics

Information on property characteristics is crucial for determining whether property that was assessed is essentially the same as what was sold. Data for both sold and unsold properties should be current, relevant, and collected in a consistent manner.

4.2 Frequency of Ratio Studies

The purpose of a ratio study dictates how often it should be conducted. Regardless of the reappraisal cycle, ratio studies made by assessors should be conducted at least annually. This frequency enables potential problems to be recognized and corrected before they become serious.

When there is a revaluation, assessors should conduct at least four ratio studies to establish the following:

1. a baseline of current appraisal performance

2. preliminary values so that any major deficiency can be corrected

3. values used in assessment notices sent to taxpayers

4. final values after completion of the first, informal phase of the appeals process

The final study can be used in planning for the following year. In addition, ratio studies can be conducted as needed to evaluate appraisal procedures, investigate a discrimination complaint, or answer a specific question.

4.3 Date of Analysis

The date of analysis depends on the purpose of the study, but generally is the assessment date of the tax year being studied, which can be the current, the next, or a past year. The assessment date of the next tax year should be used when the purpose of the study is to evaluate preliminary values in a reappraisal.

4.4 Period from Which Sales Are Drawn

This period depends on the purpose of the study and on sales activity. In general, the period should be as short as possible and, ideally, no more than one year. A longer period may be required to produce a representative sample for some strata within a jurisdiction.

To develop an adequate sample size, the sales used in ratio studies can span a period of as long as five years provided there have been no significant economic shifts or changes to property characteristics and sales prices have been adjusted for time as necessary.

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4.5 Sample Representativeness

In general, a ratio study is valid to the extent that the sample is sufficiently *representative* of the population.

The distribution of ratios in the population cannot be ascertained directly and appraisal accuracy can vary from property to property. By definition, a ratio study sample would be representative when the distribution of ratios of properties in the sample reflects the distribution of ratios of properties in the population. Representativeness is improved when the sample proportionately reflects major property characteristics present in the population of sold and unsold properties. As long as sold and unsold parcels are appraised in the same manner and the sample is otherwise representative, statistics calculated in a sales ratio study can be used to infer appraisal performance for unsold parcels.

However, if parcels that sell are selectively reappraised based on their sale prices and if such parcels are in the ratio study, uniformity inferences will not be accurate (appraisals appear more uniform than they are). In this situation, measures of appraisal level also will not be supportable unless similar unsold parcels are appraised by a model that produces the same overall percentage of market value (appraisal level) as on the parcels that sold (see Appendix E, "Sales Chasing Detection Techniques"). Assessing officials must incorporate a quality control program; including checks and audits of the data, to ensure that sold and unsold parcels are appraised at the same level.

Operationally, representativeness is improved when the following occur:

1. Appraisal procedures used to value the sample parcels are similar to procedures used to value the corresponding population

2. Accuracy of recorded property characteristics data for sold property does not differ substantially from that of unsold property,

3. Sample properties are not unduly concentrated in certain areas or types of property whose appraisal levels differ from the general level of appraisal in the population

4. Sales have been appropriately screened and validated (see Appendix A).

The first requirement generally is met unless sampled parcels are valued or updated differently from nonsampled parcels, or unless appraisals of sample parcels were done at a different time than appraisals of nonsampled parcels. For example, it is unlikely that the sample is representative of unsold parcels when the sample consists mostly of new construction, first-time sales of improved properties, condominium conversions, or newly platted lots.

The second requirement is met only if value-related property characteristics are updated uniformly for all property in a class as opposed to being updated only upon sale.

The third requirement relates to the extent to which appraisal performance for the sample reflects appraisal performance for the population.

The fourth requirement generally is met when the sales to be used in the sample are properly screened, adjusted if necessary, and validated.

4.6 Acquisition and Validation of Sales Data

Sales data are important in ratio studies and play a crucial role in any credible and efficient mass appraisal system. In some instances, it may be necessary to make adjustments to sales prices so they are more representative of the market. When there is more than one sale of the same property during a study period, only one of the transactions should be used in the ratio study. For guidelines on sales validation see Appendix A.

5. Ratio Study Statistics and Analyses

Once data have been properly collected, reviewed, assembled, and adjusted, outlier handling and statistical analysis can begin. This process involves the following steps.

1. A ratio should be calculated for each observation in the sample by dividing the appraised (or assessed) value by the sale price.

2. Graphs and exhibits can be developed that show the distribution of the ratios.

3. Exploratory data analysis, including outlier identification and screening, and tests of the hypotheses of normality may be conducted.

4. Ratio study statistics of both appraisal level and uniformity should be calculated.

5. Reliability measures should be calculated.

An example of a ratio study statistical analysis report is given in table 1-1.

5.1 Data Displays

Displays or exhibits that provide a profile or picture of ratio study data are useful for illustrating general patterns and trends, particularly to nonstatisticians. The particular form of the displays, as well as the data used (e.g., sales prices, sales ratios, and property characteristics) depends on the purposes of the particular display. Types of displays useful in ratio studies are arrays, frequency distributions, histograms, plots, and maps (Gloudemans 1999).

Graphic displays can be used to

- indicate whether a sample is sufficiently representative of the properties in a stratum

- indicate the degree of nonnormality in the distribution of ratios

- depict the overall level of appraisal

Table 1-1. Example of Ratio Study Statistical Analysis Data Analyzed

Rank of ratio of observation	Appraised value ($)	Sale Price ($)	Ratio (AV/SP)
1	48,000	138,000	0.348
2	28,800	59,250	0.486
3	78,400	157,500	0.498
4	39,840	74,400	0.535
5	68,160	114,900	0.593
6	94,400	159,000	0.594
7	67,200	111,900	0.601
8	56,960	93,000	0.612
9	87,200	138,720	0.629
10	38,240	59,700	0.641
11	96,320	146,400	0.658
12	67,680	99,000	0.684
13	32,960	47,400	0.695
14	50,560	70,500	0.717
15	61,360	78,000	0.787
16	47,360	60,000	0.789
17	58,080	69,000	0.842
18	47,040	55,500	0.848
19	136,000	154,500	0.880
20	103,200	109,500	0.942
21	59,040	60,000	0.984
22	168,000	168,000	1.000
23	128,000	124,500	1.028
24	132,000	127,500	1.035
25	160,000	150,000	1.067
26	160,000	141,000	1.135
27	200,000	171,900	1.163
28	184,000	157,500	1.168
29	160,000	129,600	1.235
30	157,200	126,000	1.248
31	99,200	77,700	1.277
32	200,000	153,000	1.307
33	64,000	48,750	1.313
34	192,000	144,000	1.333
35	190,400	141,000	1.350
36	65,440	48,000	1.363

Note: Due to rounding, totals may not add to match those on following table, which reports results of statistical analysis of above data.

Results of statistical analysis

Statistic	Result
Number of observations in sample	36
Total appraised value	$3,627,040
Total sale price	$3,964,620
Average appraised value	$100,751
Average sale price	$110,128
Mean ratio	0.900
Median ratio	0.864
Weighted mean ratio	0.915
Coefficient of dispersion (COD)	29.8%
Price-related differential (PRD)	0.98
Price-related bias (PRB) coefficient (*t*-value)	.232 (3.01)
95% median two-tailed confidence interval	(0.684, 1.067)
95% weighted mean two-tailed confidence interval	(0.806, 1.024)
Normal distribution of ratios (0.05 level of significance)	Reject— D'Agostino, Pearson K^2, and Shapiro-Wilk W
Date of analysis	9/99/9999
Category or class being analyzed	Residential

- depict the degree of uniformity
- depict the degree of value bias (regressivity or progressivity)
- compare the level of appraisal or degree of uniformity among strata
- detect outlier ratios
- identify specific opportunities to improve mass appraisal performance
- track performance measures over time

5.2 Outlier Ratios

Outlier ratios are very low or high ratios as compared with other ratios in the sample. The validity of ratio study statistics used to make inferences about population parameters could be compromised by the presence of outliers that distort the statistics computed from the sample. One extreme outlier can have a controlling influence over some statistical measures. However, some statistical measures, such as the median ratio, are resistant to the influence of outliers and trimming would not be required. Although the coefficient of dispersion (COD) is affected by extreme ratios, it is affected to a lesser extent than the coefficient of variation (COV) and the mean. The weighted mean and price-related differential (PRD) are sensitive to sales with high prices even if the ratios on higher priced sales do not appear unusual relative to other sales. Regression analysis, sometimes used in assessment ratio analyses (e.g., when ratios are regressed on sales prices or property characteristics, such as lot size or living area), is also affected by outliers: both ratio outliers and outliers based on the comparison characteristics (an excellent treatment of the assumptions made in regression and deviations from can be found in Cook, R.D. and Weisberg, S. 1982).

Outlier ratios can result from any of the following:

1. an erroneous sale price
2. a nonmarket sale
3. unusual market variability
4. a mismatch between the property sold and the property appraised
5. an error in the appraisal of an individual parcel
6. an error in the appraisal of a subgroup of parcels
7. any of a variety of transcription or data handling errors

In preparing any ratio study, outliers should be

1. identified
2. scrutinized to validate the information and correct errors
3. trimmed if necessary to improve sample representativeness

For guidelines on outlier identification and trimming, see Appendix B, "Outlier Trimming Guidelines."

5.3 Measures of Appraisal Level

Estimates of appraisal level are based on measures of central tendency. They should be calculated for each stratum and for such aggregations of strata as may be appropriate. Several common measures of appraisal level (central tendency) should be calculated in ratio studies, including the median ratio, mean ratio, and weighted mean ratio. When one of these measures is calculated on the data in a sample, the result is a point estimate, which is accurate for the sample but is only one indicator of the level of appraisal in the population. Confidence intervals around the measures of level provide indicators of the reliability of the sample statistics as predictors of the overall level of appraisal of the population. Note that noncompliance with appraisal level standards cannot be determined without the use of confidence intervals or hypothesis tests.

5.3.1 Median

The median ratio is the middle ratio when the ratios are arrayed in order of magnitude. If there is an even number of ratios, the median is the average of the two middle ratios.

The median always divides the data into two equal parts and is less affected by extreme ratios than the other measures of central tendency. Because of these properties, the median is the generally preferred measure of central tendency for evaluating overall appraisal level, determining reappraisal priorities, or evaluating the need for a reappraisal.

5.3.2 Arithmetic Mean

The arithmetic mean (aka mean or average) ratio is the average of the ratios. It is calculated by summing the ratios and dividing by the number of ratios. In a normal distribution the mean equals the median. In a distribution skewed to the right (typical of ratio study data), the mean is greater than the median. The mean is affected more by extreme ratios than the median.

5.3.3 Weighted Mean

The weighted mean ratio is the value-weighted average of the ratios in which the weights are proportional to the sales prices. The weighted mean also is the ratio of the average assessed value to the average sales price value. The weighted mean gives equal weight to each dollar of value in the sample, whereas the median and mean give equal weight to each parcel. The weighted mean is an important statistic in its own right and also is used in computing the PRD, a measure of uniformity between high- and low-value properties

The weighted mean also can be calculated by (1) summing the appraised values, (2) summing the sales prices, and (3) dividing the first result by the second. The weighted mean also is called the *aggregate ratio*.

5.3.4 Contrasting Measures of Appraisal Level

Because it gives equal weight to each ratio and is unaffected by extreme ratios, the median is the preferred measure of central tendency for evaluating appraisal performance. Although the mean ratio is also a parcel-based measure, it can be affected appreciably by extreme ratios and can be relied upon only if the sample is of adequate size and contains few outliers.

5.4 Measures of Variability

Measures of dispersion or variability relate to the uniformity of the ratios and should be calculated for each stratum in the study. In general, the smaller the measure, the better the uniformity, but extremely low measures can signal one of the following:

acceptable causes

- extremely homogeneous properties

- very stable markets

unacceptable causes

- lack of quality control

- calculation errors

- poor sample representativeness

- sales chasing

Note that as market activity changes or as the complexity of properties increases, the measures of variability usually increase, even though appraisal procedures may be equally valid.

5.4.1 Coefficient of Dispersion

The most generally useful measure of variability or uniformity is the COD. The COD measures the average percentage deviation of the ratios from the median ratio and is calculated by the following steps:

1. subtract the median from each ratio

2. take the absolute value of the calculated differences

3. sum the absolute differences

4. divide by the number of ratios to obtain *the average absolute deviation*

5. divide by the median

6. multiply by 100

The COD has the desirable feature that its interpretation does *not* depend on the assumption that the ratios are normally distributed. In general, more than half the ratios

fall within one COD of the median. The COD should not be calculated about the mean ratio.

5.4.2 Other Measures of Variability

Other useful measures of variability or the distribution of ratio study data are as follows:

- range

- percentiles

- quartiles

- interquartile range

- median absolute deviation (MAD)

- median percent deviation

- coefficient of concentration

- standard deviation

- coefficient of variation (COV)

- weighted coefficient of dispersion

- weighted coefficient of variation

See *Property Appraisal and Assessment Administration* (IAAO 1990, chapter 20) and Gloudemans (1999, chapter 5) for further discussion on these statistical measures.

Note that the typical percentage error is not the COD, but is expressed by the median percentage deviation statistic. Also, it is the interquartile range, not the COD, that brackets the middle 50 percent of the assessment ratios.

5.5 Measures of Reliability

Reliability, in a statistical sense, concerns the degree of confidence that can be placed in a calculated statistic for a sample. (For example, how precisely does the sample median ratio approximate the population median appraisal ratio?) The primary measure of importance to the local assessor is the confidence interval. A confidence interval consists of two numbers (upper and lower limits) that bracket a calculated measure of central tendency for the sample; there is a specified degree of confidence that the calculated upper and lower limits bracket the true measure of central tendency for the population. See Appendix 20-4 in *Property Appraisal and Assessment Administration* (IAAO 1990) and Appendix C for guidelines on calculating small-sample confidence intervals.

New computer-intensive statistical methods, such as the "bootstrap" (Efron and Tibshirani 1993), now enable the development of confidence interval estimates for any statistic of interest, including measures of level and uniformity.

Measures of reliability explicitly take into account the errors inherent in a sampling process. In general, these measures are tighter (better) when samples are relatively large and the uniformity of ratios is relatively good.

Measures of reliability indicate whether there is a desired degree of confidence that a given level of appraisal has *not* been achieved. This does not mean that an appraiser should tolerate measures of central tendency that fail to meet goals whenever measures of reliability are wide due to small samples, poor uniformity, or both. Such cases require either additional data for proper analysis or alternative action, such as a reappraisal, if poor uniformity is the cause. Such correction might include reappraisal, trending of strata, and respecifying or recalibrating mass appraisal models (see section 9 in this part for a discussion of ratio study standards).

5.6 Vertical Inequities

The measures of variability discussed in section 5.4 relate to "horizontal," or random, dispersion among the ratios in a stratum, regardless of the value of individual parcels. Another form of inequity can be systematic differences in the appraisal of low- and high-value properties, termed "vertical" inequities. When low-value properties are appraised at greater percentages of market value than high-value properties, assessment *regressivity* is indicated. When low-value properties are appraised at smaller percentages of market value than high-value properties, assessment *progressivity* is the result. Appraisals made for tax purposes of course should be neither regressive nor progressive.

An index statistic for measuring vertical equity is the PRD, which is calculated by dividing the mean ratio by the weighted mean ratio. This statistic should be close to 1.00. Measures considerably above 1.00 tend to indicate assessment regressivity; measures below 1.00 suggest assessment progressivity. When samples are small or the weighted mean is heavily influenced by several extreme sales prices, the PRD may not be a sufficiently reliable measure of vertical inequities. A scatter plot of ratios versus appraised values or sale prices is a useful diagnostic tool. A downward (or upward) trend to the data indicates systematic regressivity (or progressivity). Assuming representativeness, high PRDs generally indicate low appraisals on high-priced properties. If not sufficiently representative, extreme sales prices can be excluded in calculation of the PRD. Similarly, when samples are very large, the PRD may be too insensitive to show small pockets in which there is significant vertical inequity. Standards for evaluating the PRD are given in section 9.2.7 in this part. In addition, more powerful statistical tests for vertical inequities are available and should be employed to determine the significance of the indication provided by the PRD (see section 5.7 in this part and Twark, Everly and Downing [1989]).

The coefficient of price-related bias (PRB) provides a more meaningful and easily interpreted index of price-related bias than the PRD. It is obtained by regressing percentage difference from the median ratio on percentage differences in value (see Appendix D). A PRB of −.045

indicates, for example, that assessment ratios fall by 4.5% when values double and increase by 4.5% when values are halved. Like all regression coefficients, the statistical reliability of the PRB can be gauged by noting its *t*-value and related significance level, and by computing confidence intervals. In table 1-4 the PRB is -0.035 and is not statistically significant.

Unacceptable vertical inequities should be addressed through reappraisal or other corrective actions. In some cases, additional stratification can help isolate the problem. Measures of level computed for value strata should not be compared as a way of determining vertical inequity because of a boundary effect that is most pronounced in the highest and lowest strata (Schultz 1996).

5.7 Tests of Hypotheses
An appropriate test should be used whenever the purpose of a ratio study is implicitly or explicitly to test a hypothesis. A hypothesis is essentially a tentative answer to a question, such as, Are residential and commercial properties appraised at equal percentages of market value? A test is a statistical means of deciding whether the answer "yes" to such a question can be rejected at a given level of confidence. In this case, if the test leads to the conclusion that residential and commercial properties are not appraised at equal percentages of market value, some sort of corrective action on the part of assessing officials is clearly indicated.

Tests are available to determine whether the

- level of appraisal of a stratum fails to meet an established standard

- meaningful differences exist in the level of appraisal between two or more strata

- high-value properties are appraised at a different percentage of market value than low-value properties

Appropriate tests are listed in table 1-2 and discussed in Gloudemans (1999), *Property Appraisal and Assessment Administration* (IAAO 1990), and *Improving Real Property Assessment* (IAAO 1978, 137–54).

5.8 The Normal Distribution
Many conventional statistical methods assume the sample data conform to the shape of a bell curve, known as the normal (or Gaussian) distribution. Performance measures based on the mean or standard deviation can be misleading if the study sample does not meet the assumption of normality. As a first step in the analysis, the distribution of sample ratios should be examined to reveal the shape of the data and uncover any unusual features. Although ratio study samples typically do not conform to the normal distribution, graphical techniques and numerical tests can be used to explore the data thoroughly. Traditional choices are the binomial, chi-square, and Lilliefors tests. Newer and more powerful procedures are the Shapiro-Wilk W, the D'Agostino-Pearson K^2, and the Anderson-Darling A^2 tests (D'Agostino and Stephens 1986).

5.9 Parametric and Distribution-Free (Nonparametric) Statistics
For every problem that might be solved by using statistics, there is usually more than one measure or test. These measures and tests can be divided into two broad categories: parametric and distribution-free (nonparametric). Parametric statistics assume the population data conform to a known family of probability distributions (such as the normal distribution). When the mean, weighted mean, and standard deviation are used in this context, they tend to be more meaningful. Distribution-free statistics make less restrictive assumptions and do not require knowledge about the shape of the underlying population distribution. Given similar distribution of ratios in the underlying populations, distribution free tests, such as the Mann-Whitney test, can determine the likelihood that the level of assessment

Table 1-2. Tests of Hypotheses

Null Hypothesis	Nonparametric Test	Parametric Test
1. Ratios are normally distributed.	Shapiro-Wilk W test D'Agostino-Pearson K^2 test Anderson-Darling A^2 test Lillifores Test	N/A
2. The level of appraisal meets legal requirements.	Binomial test	*t*-test
3. Two property groups are appraised at equal percentages of market value.	Mann-Whitney test	*t*-test
4. Three or more property groups are appraised at equal percentages of market value.	Kruskal-Wallis test	Analysis of Variance
5. Low- or high-value properties are appraised at equal percentages of market value.	Spearman Rank test	PRB, correlation or regression analysis
6. Sold and unsold parcels are treated equally.	Mann-Whitney test	*t*-test

of property groups differ (Hart 2001). Distribution-free statistics are the median and the COD.

6. Sample Size

6.1 Importance of Sample Size

There is a general relationship between statistical reliability and the number of observations in a sample. The larger the sample size, the greater the reliability.

6.2 Adequacy of a Given Sample Size

The adequacy of a given sample size can be evaluated by computing measures of reliability. If the confidence interval is sufficiently narrow, the sample is large enough. If the confidence interval is too wide, the assessor must either accept less precision or enlarge the sample, if possible.

6.3 Required Sample Size

Formulas are available to compute the minimum sample size necessary to produce selected margins of error at a specified level of confidence. Such formulas depend crucially on the estimated variability of the ratios (Cochran 1977).

6.4 Remedies for Inadequate Samples

Small samples should be enlarged if the assessor desires to increase the reliability of statistical measures. Inadequate sample sizes are typically indicated by unacceptably wide confidence intervals. The following alternatives should be considered:

1. *Restratification.* If levels of appraisal are similar or properties are homogenous, broader strata containing larger samples can be created by combining existing strata or by stratifying on a different basis.

2. *Extending the period from which sales are drawn.* This is often the most practical and effective approach. Sales from prior years can be used; however, adjusting the sale price for time may be necessary and significant property characteristics must not change.

3. *Enlarging the sample by validating previously rejected sales.* Sales previously excluded from the analysis, because it was not administratively expedient to confirm them or to make adjustments, can be reevaluated.

4. *Imputing appraisal performance.* Ratio study statistics for strata with no or few sales can sometimes be imputed from the results obtained for other strata. These strata should be as similar as possible. Procedures and techniques used to appraise properties in the strata also should be similar.

6.5 Other Sample Size-Related Representativeness Problems

Sales from areas or substrata in which the number of sales is disproportionately large can distort ratio study results by weighting level and uniformity indicators toward whatever conditions exist in the overrepresented area. To alleviate this problem and create better representativeness, large samples can be further stratified by

- randomly selecting sales to be removed
- isolating the overrepresented groups into substrata
- redefining the time period for the overrepresented groups
- weighting the data

7. Reconciliation of Ratio Study Performance Measures

An important objective of a ratio study conducted by a local jurisdiction is the evaluation of model performance. This is a USPAP requirement in the reconciliation of a mass appraisal. Assessing officials must incorporate a quality control program, including checks and audits of the data, to ensure that sold and unsold parcels are appraised at the same level. This also requires characteristic data for both sold and unsold properties to be current, appropriate, relevant, and collected in a consistent manner.

8. Presentation of Findings, Documentation, and Training

The findings of a ratio study should be sufficiently detailed and documented to meet the needs of the users of the study. Documentation for internal ratio studies can be less detailed than for reports prepared for external uses. The following documentation should be provided in conjunction with any published ratio study.

8.1 Text

A brief text describing the purpose and the methods used should accompany a ratio study. This information can be incorporated in the report of the findings or be contained in a separate memorandum. The text should contain the statistics presented and outline the major procedural steps in completing the study. The text also should describe any rules for eliminating sales or extreme ratios and acknowledge any significant limitations in the data.

8.2 Exhibits

The body of the ratio study report should include for each stratum the statistical results intended to be used for decision-making purposes. All reports should contain the following information:

- date and tax year of the appraisals being evaluated

- number of parcels in each stratum

- number of sales

- number of sales trimmed from the study

- measures of central tendency (appraisal level)

- measures of uniformity (variability) and price-related biases

- confidence interval (measures of reliability) about the measures of central tendency

- summary of adjustments made to sales prices

In addition, there should be a description of the steps taken to ensure that sold and unsold properties were valued and described consistently. If the sold and unsold properties were not treated identically, the documentation should characterize the differences discovered between them.

8.3 Analyses and Conclusions

An objective statement of the results of the ratio study should be prepared. If the study is one in a series, a comparison of the results with those of previous studies can be helpful.

8.4 Documentation

Ratio study procedures should be documented thoroughly. This documentation should take three forms. First, a general guideline should explain the design of the study. This guideline should be updated whenever procedures are changed. Second, all software applications should be documented so that the program logic can be reviewed and modified as needed. Third, a user's manual should explain how to execute the study or run the software.

8.5 Training and Education

The effectiveness of ratio studies can be improved through education and training. Assessment supervisors should conduct seminars or workshops for the appraisal staff to explain how to interpret reports, how ratio studies can be used to improve appraisal performance, and how the results will be used in-house.

9. Ratio Study Standards

Each local jurisdiction should have ratio study performance standards. Local standards should be consistent with state or provincial standards. The standards summarized in table 1-3 are suggested for jurisdictions in which current market value is the legal basis for assessment. In general, when these standards or other local standards are not met, reappraisal or other corrective measures should be taken.

All standards recommended in this section are predicated on the assumption that steps have been taken to maximize representativeness and validity in the underlying ratio study.

9.1 Level of Appraisal

In analyzing appraisal level, ratio studies attempt to measure statistically how close appraisals are to market value (or to a required statutory constraint that can be expressed as a percentage of market value) on an overall basis. While the theoretically desired level of appraisal is 1.00, an appraisal level between 0.90 and 1.10 is considered acceptable for any class of property. However, each class of property must be within 5 percent of the overall level of appraisal of the jurisdiction (see Section 9.2.1 in this part). Both criteria must be met. By themselves, the calculated measures of central tendency provide only an indication, not proof, of whether the level meets the appropriate goal. Confidence intervals and statistical tests should be used

Table 1-3. Ratio Study Uniformity Standards indicating acceptable general quality*

Type of property—General	Type of property—Specific	COD Range**
Single-family residential (including residential condominiums)	Newer or more homogeneous areas	5.0 to 10.0
Single-family residential	Older or more heterogeneous areas	5.0 to 15.0
Other residential	Rural, seasonal, recreational, manufactured housing, 2–4 unit family housing	5.0 to 20.0
Income-producing properties	Larger areas represented by large samples	5.0 to 15.0
Income-producing properties	Smaller areas represented by smaller samples	5.0 to 20.0
Vacant land		5.0 to 25.0
Other real and personal property		Varies with local conditions

These types of property are provided for guidance only and may not represent jurisdictional requirements.

* *Appraisal level for each type of property shown should be between 0.90 and 1.10, unless stricter local standards are required.*

PRD's for each type of property should be between 0.98 and 1.03 to demonstrate vertical equity.

PRD standards are not absolute and may be less meaningful when samples are small or when wide variation in prices exist. In such cases, statistical tests of vertical equity hypotheses should be substituted (see table 1-2).

** *CODs lower than 5.0 may indicate sales chasing or non-representative samples.*

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to determine whether it can be reasonably concluded that appraisal level differs from the established goal in a particular instance. Additionally, when uniformity measures show considerable variation between ratios, level measurements may be less meaningful.

9.1.1 Purpose of Level-of-Appraisal Standard

Jurisdictions that follow the IAAO recommendation of annual revaluations *(Standard on Property Tax Policy* [IAAO 2010] and *Standard on Mass Appraisal of Real Property* [IAAO 2013]) and comply with USPAP standard rules should be able to develop mass appraisal models that maintain an overall ratio level of 100 percent (or very near thereto). However, the local assessor may be compelled to follow reappraisal cycles defined by a legal authority or public policy that can extend beyond one year. During extended cycles the influence of inflation or deflation can shift the overall ratio.

The purpose of a performance standard that allows reasonable variation from 100 percent of market value is to recognize uncontrollable sampling error and the limiting conditions that may constrain the degree of accuracy that is possible and cost-effective within an assessment jurisdiction. Further, the effect of performance standards on local assessors must be considered in light of public policy and resources available.

9.1.2 Confidence Intervals in Conjunction with Performance Standards

The purpose of confidence intervals and similar statistical tests is to determine whether it can be reasonably concluded that the appraisal level differs from the established performance standard in a particular instance. A conclusion of noncompliance requires a high degree of confidence; thus, a 90 percent (two-tailed) or 95 percent (one-tailed) confidence level should be used, except for small or highly variable samples. The demonstration ratio study report in table 1-4 presents 95% two-tailed confidence interval estimates for the mean, median, and weighted mean ratio.

9.2 Appraisal Uniformity

Assuming the existence of an adequate and sufficiently representative sample, if the uniformity of appraisal is unacceptable, model recalibration and/or reappraisal should be undertaken. It is important to recognize that the COD is a point estimate and, especially for small samples, should not be accepted as proof of assessment uniformity problems. Proof can be provided by recognized statistical tests, including bootstrap confidence intervals.

In unusually homogeneous strata, low CODs can be anticipated. In all other cases, CODs less than 5 percent should be considered suspect and possibly indicative of nonrepresentative samples or selective reappraisal of selling parcels.

9.2.1 Uniformity among Strata

Although the goal is to achieve an overall level of appraisal equal to 100 percent of the legal requirement, ensuring uniformity in appraisal levels among strata also is important. The level of appraisal of each stratum (class, neighborhood, age group, market areas, and the like) should be within 5 percent of the overall level of appraisal of the jurisdiction. For example, if the overall level of appraisal of the jurisdiction is 1.00, but the appraisal

Table 1-4. Demonstration Ratio Study Report

Rank	Parcel #	Appraised value	Sale price*	Ratio	Statistic	Result
1	9	$87,200	138,720	0.629	Number (n)	17
2	10	38,240	59,700	0.641	Total appraised value	$1,455,330
3	11	96,320	146,400	0.658	Total sale price	$1,718,220
4	12	68,610	99,000	0.693	Avg appraised value	$85,608
5	13	32,960	47,400	0.695	Avg sale price	$101,072
6	14	50,560	70,500	0.717		
7	15	61,360	78,000	0.787	Mean ratio	0.827
8	16	47,360	60,000	0.789	Median ratio	0.820
9	17	56,580	69,000	0.820	Weighted mean ratio	0.847
10	18	47,040	55,500	0.848		
11	19	136,000	154,500	0.880	Coefficient of dispersion	14.5
12	20	98,000	109,500	0.895	Price-related differential	0.98
13	21	56,000	60,000	0.933	PRB	−0.035
14	22	159,100	168,000	0.947	PRB coefficient (*t*-value)	0.135 (2.4)
15	23	128,000	124,500	1.028		
16	24	132,000	127,500	1.035	95% conf. int. mean (two-tailed)	0.754 to 0.901
17	25	160,000	150,000	1.067	95% conf. int. median (two-tailed)	0.695 to 0.933
					95% conf. int. wtd. mean (two-tailed)	0.759 to 0.935

Date: 0/0/00.   *No outlier trimming*

* or adjusted sale price

level for residential property is 0.93 and the appraisal level for commercial property is 1.06, the jurisdiction is not in compliance with this requirement. This test should be applied only to strata subject to compliance testing. It can be concluded that this standard has been met if 95 percent (two-tailed) confidence intervals about the chosen measures of central tendency for each of the strata fall within 5 percent of the overall level of appraisal calculated for the jurisdiction. Using the above example, if the upper confidence limit for the level of residential property is 0.97 and the lower confidence limit for commercial property is 1.01, the two strata are within the acceptable range.

9.2.2 Uniformity among Single-Family Residential Properties

The COD for single-family homes and condominiums in older or more heterogeneous areas should be between 5.0 and 15.0. In areas of newer or fairly similar residences, it should be between 5.0 and 10.0.

9.2.3 Uniformity among Income-Producing Properties

The COD should be between 5.0 and 20.0. In larger, urban market areas, it should be between 5.0 and 15.0.

9.2.4 Uniformity among Unimproved Properties

The COD for vacant land should be between 5.0 and 20.0. The upper limit for an acceptable COD for vacant rural residential or seasonal land may be 25.0.

9.2.5 Uniformity among Rural Residential and Seasonal Properties, Manufactured Housing, and Multifamily Dwellings

The COD for heterogeneous rural residential property, recreational or seasonal homes, manufactured housing, and multifamily dwellings (2-4 units) should be between 5.0 and 20.0.

9.2.6 Uniformity among Other Properties

Target CODs for special-purpose real property and personal property should reflect the nature of the properties involved, market conditions, and the availability of reliable market indicators.

9.2.7 Vertical Equity

PRDs should be between 0.98 and 1.03. The reason this range is not centered on 1.00 relates to an inherent upward bias in the arithmetic mean (numerator in the PRD) that does not equally affect the weighted mean (denominator in the PRD). When samples are small, have high disper-

sion, or include properties with extreme values, the PRD may not provide an accurate indication of assessment regressivity or progressivity. When relying on the PRD to measure vertical equity, it is good practice to perform an appropriate statistical test for price-related biases before concluding that they exist (see table 1-2).

The PRB provides a measure of price-related bias that is more meaningful and less sensitive to extreme prices or ratios. As a general matter, the PRB coefficient should fall between −0.05 and 0.05. PRBs for which 95% confidence intervals fall outside of this range indicate that one can reasonably conclude that assessment levels change by more than 5% when values are halved or doubled. PRBs for which 95% confidence intervals fall outside the range of −0.10 to 0.10 indicate unacceptable vertical inequities.

As an illustration of the above, assume that the PRB is −.115 with a standard error of 0.02 and corresponding 95% confidence interval of −0.075 to −0.155 (−0.115 ± 0.04 approximately). One can conclude with 95% confidence that assessment levels change by at least 7.5% when values double or are halved but not that assessment levels change by at least 10%. This result would not be out of compliance with the ± 0.10 standard.

9.2.8 Alternative Uniformity Standards

The above standards may not be applicable to properties in unique, depressed, or rapidly changing markets. In such cases, assessment administrators may be able to develop target standards based on an analysis of past performance or results in similar markets elsewhere. Such an analysis can be based on ratio study results for the past five years or more.

9.3 Natural Disasters and Ratio Study Standards

Natural disasters such as earthquakes, floods, and hurricanes can have a substantial impact on the interpretation and use of ratio studies. In particular, they

- increase the difficulty of accurately identifying the physical and economic characteristics of property on the dates of sale and appraisal

- increase the difficulty of producing sufficiently reliable appraised values

- decrease the availability of usable sales and other market data

- disrupt the supply and demand equilibrium in the neighborhood community or region

As a result of these potential problems, a number of unreliable sample properties may need to be excluded and sample sizes may be unavoidably reduced. All these factors should be considered when ratio study standards are being