

applied to study results from areas substantially affected by disasters. Such consideration must not result in unwarranted relaxation of applicable standards. When faced with such situations, assessors must use informed, reasoned judgment and common sense to produce a sufficiently reliable ratio study, based upon the best information available.

10. Personal Property Ratio Studies

Studies can be done by local assessors to determine the quality of assessments of personal property in their jurisdictions. For guidelines on conducting personal property ratio studies, see section 12 in Part 2.

Standard on Ratio Studies

Part 2. Equalization and Performance Monitoring

1. Scope

This part of the standard provides guidance and supplementary information to oversight agencies that perform ratio studies. Oversight or equalization ratio studies are designed to examine the overall degree of accuracy of assessments within or among categories of property, market areas, assessment jurisdictions or political subdivisions, such as school districts, municipalities, counties, states or provinces.

2. Oversight Ratio Studies

Oversight agencies are often required to monitor appraisal performance and take corrective actions when necessary. Equalization is a common tool used by oversight agencies to address problems associated with appraisal level. Reappraisal orders can be used to correct uniformity problems.

2.1 Monitoring of Appraisal Performance

Oversight agencies usually perform sales ratio studies, which can include independent appraisals, to monitor local assessment performance. The findings can serve as the basis for enforcement actions, such as reappraisal or equalization orders. State/provincial agencies also often perform ratio studies to advise assessors and the public about local appraisal conditions. Many state or provincial oversight agencies have a dual role. One role is to advise and assist local appraisal offices, and the other role is to measure local appraisal performance. These two roles can create a conflict of interest, which should be minimized.

2.2 Equalization

Oversight agencies can use the results of ratio studies to equalize, directly or indirectly, appraisals or assessments in taxing jurisdictions. Direct equalization is accomplished by an oversight agency which alters locally determined assessments by ordering appraisals within jurisdictions or property classes to be adjusted to market value or to the legally required level of assessment. Direct equalization can also involve adjusting appraisals of centrally assessed properties. When indirect equalization is used, appraisals are not adjusted. Instead, indirect equalization involves an oversight agency estimating total taxable value, given the legally required level of assessment or market value. Indirect equalization allows proper distribution of intergovernmental transfer payments between state or provincial and local governments despite different levels of appraisal among

jurisdictions or property classes. Equalization is not an appraisal or a substitute for reappraisal.

When equalization is based on ratio study samples, sampling error must be taken into account. When confidence intervals include an acceptable range, equalization cannot be supported statistically. When confidence intervals *fail* to bracket official requirements, equalization actions are supported (see section 6.5, “Measures of Reliability,” and section 11.1, “Level of Appraisal”).

Legal aspects of ratio studies, many of which relate to equalization, are discussed in Appendix G.

2.2.1 Direct Equalization

Many states and provinces have authority and specific procedures for direct equalization. The advantage of direct equalization is that it can be applied to specified strata, such as property classes, geographic areas, and political subdivisions that fail to meet appraisal level performance standards (Dornfest [Journal of Property Tax Assessment and Administration, 2004]). Direct equalization also produces results that are generally more visible to the taxpayer and more clearly reduces perceived inequities between classes (*Standard on Property Tax Policy* [IAAO 2010]). For example, direct equalization allows proper and equal application of debt and tax rate limits and equitable partial exemptions.

Direct equalization involves use of adjustment factors, which produce effects mathematically identical to those derived through the application of “trending” or “index” factors, which are commonly used for value updating by local assessing jurisdictions. The most significant differences typically are the level of the jurisdiction originating the adjustments and the stratification of property to which the factors are applied. Local jurisdictions with primary assessment responsibility can develop value adjustment factors as an interim step between complete reappraisals. Such factors commonly are applied to properties by property type, location, size, age and other characteristics (see *Property Appraisal and Assessment Administration* [IAAO 1990, p. 310]). It is rare for equalization factors developed by oversight agencies to be applied to strata more specific than property class or broad geographic area. Often such factors are applied jurisdiction-wide.

States and provinces that employ direct equalization techniques should understand that such equalization is not a substitute for appraisal or reappraisal. Direct equalization

applied at the stratum level improves equality in effective tax rates between strata and lessens the effect of assessment practices that improperly favor one stratum over another. For example, assuming that all classes of property are to be assessed at 100% of market value, without such equalization, in a case where residential property is assessed at a median of 80% of market value, while commercial property is assessed at a median of 90% of market value, residential property will pay 80% of its proper tax share and commercial property will pay 90% of its proper tax share. Other classes that may be assessed at 100% will pay more than their proper tax shares. Direct equalization mitigates this problem. However, such equalization cannot improve uniformity between properties within a given stratum. So, in the previous example, the median level of assessment for residential property can be adjusted from 80% to 100% of market value, assessment disparities between individual residential properties will not be addressed. For this reason, reappraisal orders should be considered as the primary corrective tool for uniformity problems, and direct equalization should be considered appropriate only if time or other constraints preclude such an approach.

2.2.2 Indirect Equalization

The most common use of indirect equalization is to enable proper funding distribution, particularly for school districts. Such equalization provides an estimation of the proper tax base (acknowledging statutory constraints such as agricultural use value) despite appraisals that are higher or lower than legally required levels in certain jurisdictions. For example, if the assessed value of residential property in a jurisdiction is \$750 million, but a residential ratio study shows an assessment level of 75 percent, while the legally required level of assessment is 100 percent, an equalized value of \$1,000 million could be computed ($\$750 \text{ million} / 0.75$). This adjusted or equalized value would then be used to apportion payments or requisitions between the state or province and associated local governments.

Indirect equalization results in fairer funding apportionment because the overall appraisal levels of the taxing jurisdictions tend to vary. If there were no equalization, the extent that a jurisdiction under- or overestimated its total tax base would result in over- or under-apportionment of funds. Indirect equalization does not correct under- or overvaluation between classes of property within a jurisdiction. It adjusts only a portion of the tax or sometimes only intergovernmental payments, is less visible to taxpayers, and often lacks checks and balances associated with direct equalization (see *Standard on Property Tax Policy* [IAAO 2010]). By adjusting governmental payments, tax rates, or partial exemptions, indirect equalization encourages taxing jurisdictions to keep their overall tax bases close to the required level.

Whether used to equalize shared funding or tax rates, the degree of equalization of the property tax is more limited than with direct equalization. Indirect equalization generally is applied to or affects only a portion of the funding or property tax levy (perhaps the school general levy or city levy). Indirect equalization usually is applied to the jurisdiction, rather than to a stratum, and therefore resolves interjurisdictional discrepancies in assessment level. In addition, properties in strata with poor uniformity are affected disproportionately. For this reason, indirect equalization also is not a substitute for reappraisal.

3. Steps in Ratio Studies

Ratio studies conducted by oversight agencies generally follow the basic steps described for the assessor's office in Part 1, except that it is more important to adopt uniform procedures and be consistent in their application.

3.1 Definition of the Purpose, Scope, and Objectives

The first step in any ratio study is to determine and state clearly the reasons for the study. This crucial step of identifying the purpose of the study determines the specific goals, scope, content, depth, and required flexibility.

3.2 Design of Study

The most important design consideration is that the study sample be sufficiently representative of the population of properties or the distribution of values in the jurisdiction under review. For direct equalization the level of appraisal for property classes or strata subject to such equalization is the primary area of interest and the sample must be designed accordingly. Indirect equalization seeks to estimate the overall dollar value of the population, so the sample must be representative of that overall value and must reflect the disproportionate influences of high value properties. Performance monitoring is concerned with both level and uniformity, but typically involves sample design similar to that required in direct equalization.

3.2.1 Level of Sophistication and Detail

A basic design principle is to keep the study as simple as possible consistent with its purpose. Ratio studies are not all alike and should be tailored to an intended use.

Data analysis has been made easier through computerization. Although every study does not require the same level of statistical detail, each ratio study should include measures of appraisal level, appraisal uniformity, and statistical reliability. Graphs, charts, or other pictorial representations can be useful tools for showing distributions and patterns in the data. There is no model ratio study design that can serve all jurisdictions or all situations equally well. Informed, reasoned judgment and common sense are required in the design of ratio studies.

3.2.2 Sampling

A ratio study is a form of applied statistics, because the analyst draws conclusions about the appraisal of the universe (the entire jurisdiction) of properties based only on those that have sold during a given time period or appraisals selected for a random sample. The ratios constitute the sample that will be used to draw conclusions or inferences about the population.

To determine the accuracy of appraisals within a jurisdiction with absolute certainty, it would be necessary for all properties in the population to have been sold in arm's-length, open-market transfers near the appraisal date or all properties would need to be appraised independently by the oversight agency. Since this is not possible, ratio studies must use samples and draw inferences or conclusions about the population from these samples.

The number of parcels in the population (the jurisdiction or stratum) is not an important determinant of a statistically valid and reliable sample.

3.2.3 Determining the Composition of Samples

In the design stage, the oversight agency must decide whether the ratio study sample should comprise sales (or asking prices when appropriate), independent appraisals, or a combination of the two. Each sample type has its advantages and disadvantages, as described below.

3.2.3.1 Sale Samples

The advantages of using sale samples include the following:

- Properly validated sales provide more objective indicators of market value than independent appraisals.
- Using sales is much less expensive than producing independent appraisals.

The disadvantages include the following:

- Difficulty in collecting sales data in jurisdictions without disclosure documents
- The oversight authority may not have control over the sales data collection and validation process
- Influence of sales chasing can be difficult to detect or prevent
- Samples of sales may not adequately represent the population of properties
- An adequate sample size may not be achieved if sales data are scarce
- Time adjustments are more critical when supplemental sales are included

3.2.3.2 Independent Appraisal Samples

Independent appraisals also can be used instead of or in addition to sales for ratio study samples. (See section 8, "Appraisal Ratio Studies," in this part.)

3.2.3.3 Samples Combining Sales and Independent Appraisals

The oversight agency can design and conduct ratio studies using samples comprised of sales and independent appraisals. In this approach, the combined advantages of sale samples and appraisal samples are realized. However, the disadvantage of combining sales and independent appraisals is the possible existence of some of the disadvantages of sale samples and/or appraisal samples (see Section 8.7).

3.3 Collection and Preparation of Market Data

The reliability of a ratio study depends in part on how accurately the sales or independent appraisals used in the study reflect market values. For sales-based studies, oversight agencies should conduct an independent sales verification and screening program if resources permit. Alternatively, oversight agencies should develop audit criteria to review data submitted to qualify sales, corroborate representativeness and confirm adequate sample size. Audit decisions should accommodate needs of the agency and resources available. Independent appraisals used in ratio studies must comply with the appropriate sections of the *Uniform Standards of Professional Appraisal Practice* (USPAP; Appraisal Foundation 2010–2011), and reflect market values as of the date being studied. Most oversight agencies use property data collected by the local jurisdiction to develop their independent appraisals. In order to produce credible appraisals, the oversight agency must be certain that the local jurisdiction accurately recorded the appropriate value-related property characteristics for each property it is independently appraising. Steps must be taken to ensure that errors in the database made by the local jurisdiction do not materially or significantly affect the conclusions or opinions of value developed by the oversight agency.

3.4 Stratification

Stratification divides all the properties within the scope of the study into two or more groups or strata. Stratification facilitates a more complete and detailed picture of appraisal performance and can enhance sample representativeness

Each type of property subject to a distinct level of assessment could constitute a stratum. Other property groups, such as market areas, school districts and tax units, could constitute additional strata.

Strata should be chosen to be consistent with factors in the mass appraisal model. When the purpose of the study is to evaluate appraisal quality, flexibility in stratification

is essential. The general goal is to identify areas in which the assessment levels are too low or lack uniformity and property groups for which additional reappraisal work may be required. In such cases, it also is highly desirable to stratify on the basis of more than one characteristic simultaneously.

Stratification can help identify differences in level of appraisal between property groups. In large jurisdictions, stratification by market areas is generally more appropriate for residential properties, while stratification of commercial properties by either geographic area or property subtypes (e.g., office, retail, and warehouse/industrial) can be more effective.

3.5 Matching Appraisal Data and Market Data

The physical and legal characteristics of each property used in the ratio study must be the same when appraised for tax purposes and when sold. This implies two essential steps. First, the property description for the sold parcel must match the appraised parcel. If a parcel is split between the appraisal date and the sale date, a sale of any of its parts should not be used in the ratio study.

Second, the property rights transferred, permitted use, and physical characteristics of the property on the date of assessment must be the same as those on the date of sale. Properties with significant differences in these factors should be excluded from the ratio study.

When statutory constraints are imposed on appraisal methods, the resulting assessment may be less than market value. In such cases a sales ratio study may not provide useful performance information. Constraints typically apply to land that qualifies for agricultural-use value, subsidized housing, mineral land, and timberland.

Sales may include property of a type other than the type for which the ratio study analyses is intended. However, sales including more than minimal values of secondary categories are unlikely to be representative, even with adjustment.

For example, a property that is predominantly commercial may include residential components. This sale can be included as representative of the commercial category. In this case, the numerator in the ratio calculation would be the total appraised value including the value of both the commercial and residential components.

In a second example, for a ratio study of vacant land, the numerator in the ratio should reflect only the appraised value of the land. The sale price should be adjusted for the contributory value of the improvements or the sample should be excluded from further analysis.

3.5.1 Stratification for Equalization Studies

Oversight agencies generally should define the strata prior to acquiring and compiling data for the ratio study.

Predefined stratification is more transparent and enhances cooperation between the oversight agency and the jurisdiction appraising the property subject to equalization. In general, oversight agencies should not redefine the strata once they have been defined for equalization purposes, especially in the case of direct equalization. It is appropriate, however, to collapse strata to compensate for otherwise inadequate samples sizes. In addition, a reappraisal or equalization order can be targeted for specific problem areas that cause noncompliance at a broader level of aggregation. If value stratification is necessary, predefined strata may not be practical.

3.5.2 Stratification for Direct Equalization

Strata should be chosen consistent with operational requirements for the required level of equalization. Statistical issues in the determination of strata include the size of the population and resulting strata and the likely variability of the ratios in each stratum. Care must be taken not to over-stratify, that is, to create strata that are too small to achieve statistical reliability (see section 6, Sample Size” in part 1 and Sherrill and Whorton [1991]). No conclusion about stratum level or uniformity should be made from stratum samples that are unreliably small (resulting in unacceptably large margins of error). Ultimately, the degree of stratification is determined largely by available sales data, unless it is cost-effective and practical to add sufficient independent appraisals. If sufficient sales or appraisals are not available for a given stratum, it should be combined with similar strata. When strata are combined, provided there is no reason to suspect dissimilar ratios as evidenced by different level or uniformity measures, such combinations permit broader applicability of ratio study results and prevent ratio study analysis from becoming too focused on substrata with few sales or appraisals. When jurisdiction or category wide equalization actions are required, reliability of component strata is not an issue.

3.5.3 Stratification for Indirect Equalization

Indirect equalization develops an estimate of full market value, but assessed values of individual properties are not altered. Such studies can use a substantially different approach to stratification than ratio studies intended for performance evaluation or direct equalization. The purpose of stratification in this case is to minimize distortions due to different assessment levels, which can vary by property type, value range, geographic area, and other factors. If stratification creates a more representative sample, equalization decisions may be based on results from individual stratum. . If the overall sample is representative of the population then equalization decisions should be based on overall sample results. A reasonable number of strata with small samples and larger margins of error can increase overall representativeness and may reduce the margin of error for the overall jurisdiction-wide sample.

The primary level of stratification should ordinarily be by major property type (e.g., residential, commercial, and vacant land). If circumstances permit, a secondary level of stratification also is recommended. When relying on the weighted mean, the secondary level of stratification (substrata) should normally be value range. Higher-value properties can sell with a different frequency than low-value properties, and appraisal levels can vary between high and low-value properties. As a result, high-value properties can be oversampled (or undersampled) and, because of their high value, can exert a disproportionate influence on the weighted mean and resulting estimated value. Value stratification reduces distortion of the weighted mean caused by over or under-representation of value strata with different levels of appraisal. To properly develop and use value strata, the oversight agency needs each individual assessment in the study universe. If detailed value information is not available, the oversight agency should work with local taxing jurisdictions to obtain sufficient information. At a minimum, a questionnaire can be used to request the total value and number of parcels in predetermined value categories or quantiles (each range contains the same amount of value).

In situations in which value stratification information is not available, or where property ratios are not significantly value-influenced, substrata can be created based on property subtype, geographic area, or other appropriate criteria. Stratification by these criteria corrects for differences in level of appraisal between substrata. In large jurisdictions, substratification by geographic areas generally is more appropriate for residential properties while sub-stratification by either geographic area or property subtypes (e.g., office, retail, and warehouse/industrial) can be appropriate for income-producing properties.

When relying on the median and when sample sizes permit, it is appropriate to stratify within property class by whichever property characteristic is most likely to capture differences in appraisal levels. This characteristic can be geographic area, property subtype, or value range. Substratification by value range helps capture value-related differences in assessment levels, which (unlike the weighted mean) are not reflected in the median.

3.6 Statistical Analysis

When ratio studies are conducted for equalization purposes, confidence intervals and statistical tests can be used to determine whether it should be concluded at a given confidence level that appraisal performance or level requirements in a stratum (or jurisdiction) being tested meets or falls outside of mandated standards. Statistical tests can be used for comparisons among strata, provided the sample sizes are large enough that meaningful differences are not missed (see section 6, “Ratio Study Statistics and Analyses”).

3.7 Evaluation and Use of Results

Lack of independence between locally determined values and sale prices (sales chasing) or independent appraisals can subvert attempts to improve equity (direct equalization) and result in incorrect distribution of funds between states or provinces and local jurisdictions (indirect equalization). To guard against these possibilities, oversight agencies should ensure that sold and unsold properties are appraised similarly. Also, appraisals used as substitutes for sales must reflect market value, and the oversight agency must take remedial measures in instances in which they do not (see section 9, “Estimating Performance of Unsold Properties”, and Appendix E, “Sales Chasing Detection Techniques”).

4. Timing and Sample Selection

Ratio studies made by oversight and equalization agencies should be conducted at least annually. Where possible, ratio studies conducted by equalization agencies should use final values established at the local level, inclusive of changes made by local appeal boards up to that time. However, if local appraisers or boards “chase sales” or set values in a manner that is dissimilar to the way other property values have been set, the sample may not be sufficiently representative and should not be used without careful investigation and necessary adjustment.

4.1 Date of Analysis

The date of analysis is a past year when appraisals from past years are being evaluated to avoid the effects of sales chasing. When prior-year assessments are used to gauge current performance (to avoid sales chasing), the results should be adjusted for any reappraisal activity or assessment changes that occurred in the population (net of new construction) between the prior and current years. Sale prices also should be adjusted to the assessment date to account for time trending.

If the purpose of the study is equalization, using sales after the appraisal date (adjusted for time as necessary) helps ensure the independence of appraisals and sales prices. A sales period spanning the appraisal date can be used if measures are taken to ensure the independence of appraisals made after the earlier sales. This approach has the advantage of reducing the importance of time adjustments.

4.2 Representativeness of Samples

The design and conduct of ratio studies requires decisions that maximize representativeness within the constraints of available resources.

In many kinds of statistical studies, samples are selected randomly from the population and from within each stratum to maximize representativeness. Ratio study samples based on independent appraisals can be randomly selected. Because sales are convenience samples and do not repre-

sent true random samples, care must be taken to maximize the representativeness of sales samples.

A ratio study sample is considered sufficiently representative for direct equalization and mass appraisal performance evaluation when the distribution of ratios of properties in the sample reflects the distribution of ratios of properties in the population. A ratio study is considered sufficiently representative for indirect equalization when the distribution of ratios of dollars of property value in the samples reflects the distribution of ratios of dollars of property value in the population.

Sales from areas or substrata in which the number of sales is disproportionately large can distort ratio study results by weighting level and uniformity indicators toward whatever conditions exist in the overrepresented area. To alleviate this problem and create better representativeness, large samples can be further stratified by

- randomly selecting sales to be removed
- isolating the overrepresented groups into substrata
- redefining the time period for the overrepresented groups
- weighting the data

4.2.1 Maximizing Representativeness with Independent Appraisals

For independent appraisal-based ratio studies, the application of random sampling techniques can help ensure that appraisal procedures used for the sampled properties are similar to the corresponding population. A well-designed random sampling plan also can help ensure that properties selected for independent appraisals are not concentrated in areas of high sales activity or associated with property types with higher turnover rates in the market.

The USPAP competency rule requires appraisers to have both knowledge and experience required to perform specific appraisals. Independent single-property appraisals must be developed in compliance with Standard 1, must be reported in compliance with Standard 2, and must be reviewed in compliance with Standard 3 of USPAP. Most importantly, care must be taken to ensure that independent appraisals reflect market value as of the appraisal date. Independent mass appraisals must be developed and reported in compliance with Standard 6 of *USPAP*.

4.2.2 Very High-Value Properties

Assessment jurisdictions often contain unique, very-high-value properties (for example, properties that constitute more than 10 percent of the value of a property class) that cannot reasonably be combined with other properties for purposes of the ratio study. For indirect equalization, high-value parcels are especially important to maximize representativeness. For instance, consider a population

consisting of 1,000 properties, 999 of which range in value from \$20,000 to \$750,000, and one that is valued at \$1 billion (e.g., a power plant). If the intended use of the ratio study is to estimate the general level and uniformity of appraisal in regard to the typical property, the stratified population of parcels need not include the \$1 billion property. If the intended use of the ratio study is to estimate the total market value in the jurisdiction, however, exclusion of the power plant can distort the study.

Very high-value properties should not be ignored or assumed to be appraised at the legal or general level for indirect equalization studies. An equalization agency should place very high-value property in a separate stratum to prevent distortion of the overall weighted mean or total estimated value. To value the property for ratio study purposes the equalization agency should use a recent properly adjusted sales price if available. If a recent sale is not available the agency should conduct an appraisal of such properties (this is the preferred option) or audit and adjust as necessary the values developed by the local jurisdiction.

5. Acquisition and Analysis of Sales Data

The highest level of independence and objectivity in an equalization or performance monitoring ratio study requires independent sales validation. If resources are not available to achieve this level of sophistication, then a comprehensive audit program should be developed to review the validation and screening work of the local jurisdiction (see Appendix A, “Sales validation Guidelines”).

5.1 Sale Adjustments for Statutorily Imposed Value Constraints

Most states and provinces require appraisal of certain classes of property using statutorily prescribed methods of appraisal that are intended to produce a constrained value that is less than market value. The most common class of property to which such constraints apply is farmland and rangeland that qualifies for agricultural-use valuation. However, constraints may also apply to subsidized housing, mineral land, and other classes. When the purpose of the ratio study is direct or indirect equalization, sales prices must be adjusted as if the selling parcel were subject to the same constraints. If this cannot be done, independent appraisals, which employ the required constraints, should be used to determine the level of appraisal in a manner consistent with the statutory constraints. For example, assume that statutory restrictions require a fixed or artificially high capitalization rate to be used in determining farmland value. If unadjusted farmland sales were to be used, the resulting ratios would be low and could lead to improper equalization decisions. Instead, independent appraisals using the required capitalization rate should be done. These appraisals would lead to ratios that would correctly allow for the statutory constraint.

Use of constrained values produces ratio study results that do not provide information on the true level of appraisal in relation to market value. Use of constrained values is appropriate for equalization. However, when the purpose of the ratio study is to determine the overall quality of assessments or the amount of benefit being awarded by a given statutory constraint on appraised value, the unadjusted sale price or independent market value appraisal must be used. Often, procedural audits can be used as adjuncts to more traditional ratio studies. These audits can be particularly effective when the purpose is to judge overall appraisal quality and when precise, quantitative statistical measures are not obtainable.

5.2 Outlier Ratios

Oversight agencies should consider the extent of sales verification when developing guidelines for trimming limits. In practice, this means that if an oversight agency derives sales data from assessing jurisdictions that may have already removed outliers from the sample, additional trimming may not be necessary (see Appendix B, “Outlier Trimming Guidelines”).

5.2.1 Value Outliers

When the weighted mean is used for indirect equalization, a method that identifies high-value *influential* sales is recommended. Since an influential sale may not have an unusually low or high ratio relative to the rest of the sample, the definition of distortion is based on the principle that the point estimate calculated from the sample should not be statistically significantly different whether the suspect observation is in the sample or not.

To test for an influential sale, one approach is to remove it from the sample and compute the weighted mean and associated confidence interval. If the weighted mean of the sample lies outside the confidence interval calculated without the influential sale, then the sale is truly influential and is a candidate for further scrutiny, isolation in a separate stratum, or possible trimming.

This procedure is intended to test the presence of individual influential sales and is not intended to be used successively after deletion of a sale, but can be applied to more than one apparent outlier at a time by leaving all other sales in the comparison group. Note, however, that the presence of multiple influential sales can indicate the start of a trend. Presence of influential sales is often associated with high price-related differential (PRD) values, which could be the result of systematic regressivity or progressivity. In contrast, the coefficient of price-related bias (PRB) is much less influenced by value outliers and should not be relied on to help identify these outliers.

5.2.2 Outlier Trimming

Statistics calculated from trimmed distributions, obviously, cannot be compared to those from untrimmed distributions or interpreted in the same way. This is especially problematic when making interjurisdictional comparisons. For this reason, oversight agencies may wish to promulgate uniform trimming procedures, based on sound statistical principles. Regardless of the chosen procedure, trimming of outliers must not occur more than once for any sample.

6. Ratio Study Statistics and Analyses

Ratio study measures covered in Part 1 are equally applicable to equalization ratio studies based upon sales or independent appraisals. See section 5.3, “Measures of Appraisal Level,” and section 5.4, “Measures of Variability,” in Part 1.

6.1 Measures of Appraisal Level

The median is the generally preferred measure of central tendency for direct equalization, monitoring of appraisal performance, or evaluation of the need for a reappraisal. The mean should not be used for indirect equalization if there are measurable differences in appraisal level of high- and low-value properties (see table 2-2). In data commonly containing outliers, the trimmed mean can be substituted for the mean (Gloude-mans 1999, chapter 3). See Appendix B for outlier-trimming procedures. Because of its dollar-weighting feature, the weighted mean is most appropriately used in indirect equalization, when estimating the total dollar value of the jurisdiction. When relying on the measure, however, outliers should be carefully reviewed (and deleted if appropriate), since they can strongly affect the weighted mean, particularly when they occur for high-value properties and in small samples.

6.2 Overall Ratio for Combined Strata

For purposes of oversight monitoring of overall appraisal performance and direct equalization, the generally preferred approach is to weight the median ratio of each stratum on the basis of the relative number of properties in the stratum. For indirect equalization, the weight assigned to a measure of central tendency of a stratum should be proportional to the share of that stratum’s total estimated market value. Because the number of parcels bears only a loose relationship to dollar value, weighting by number of parcels is not appropriate for indirect equalization.

For indirect equalization, the preferred method of calculating the overall market value of a jurisdiction is as follows:

1. Divide the total appraised (or assessed) value of each stratum by the stratum sample’s measure of

Table 2-1. Illustration of Combining Measures of Central Tendency (Example shown is for indirect equalization)

Data for properties in the study					
Stratum (1)	Total sample assessed value (2)	Total sample sale price (3)	Weighted mean (2)/(3) (4)	Total assessed value of stratum (5)	Indicated market value of stratum (6)
Residential	\$3,000,000	\$4,000,000	0.750	\$600,000,000	\$800,000,000
All other	950,000	1,000,000	0.950	400,000,000	421,000,000
Total				\$1,000,000,000	\$1,221,000,000

Overall ratio = \$1,000,000,000/\$1,221,000,000 = 0.819

Table 2-2. Preferred Estimators

	Indirect Equalization	Direct Equalization	Monitoring Performance
Median	—	X	X
Mean	—	—	—
Weighted Mean	X*	—	—

* Caution should be exercised when the sample contains value outliers or indicates value bias based on the PRD

central tendency (see section 6.3, “Contrasting Measures of Appraisal Level,” in this part) to obtain an estimate of the total market value of taxable property in the stratum.

- Sum the estimates of total stratum market value to obtain an estimate of the total market value of taxable property in the jurisdiction or class of property.
- To obtain an overall weighted level of assessment (or ratio), divide the total appraised (or assessed) value of the jurisdiction or class of property by the estimated total market value (table 2-1 contains a simplified example).

6.3 Contrasting Measures of Appraisal Level

Table 2-2 summarizes the preferred measures of central tendency for the three broad purposes of indirect equalization, direct equalization, and the general monitoring of appraisal performance.

For indirect equalization, the preferred measure is the weighted mean (the measure used in table 2-1), because it gives equal weight to each dollar. This helps achieve an accurate estimate of total dollar value, the goal of indirect equalization. However, there are implicit difficulties in obtaining sales samples that are representative of all significant groups of properties with different ratios. The weighted mean can be disproportionately influenced by high-value properties, particularly in a small sales sample. A disproportionate influence of high-value properties can be reduced through value stratification within the property class. Such value stratification helps capture value-related ratio differences, as well as improve representativeness, regardless of which measure of central tendency is used. If there are provable value-related ratio differences within strata, the weighted mean must be used since the median is incapable of capturing value-related differences. In cases

in which value stratification is not practicable, equalization agencies may stratify by some proxy for value, such as neighborhood or property sub-class. If results appear distorted by non-representative high-value sales, outlier identification methods described in Appendix B should be employed.

While not conceptually preferred, the median can be used to prevent the disproportionate influence of high-value properties with outlier ratios. To be clear, although the median is not the conceptually appropriate measure, it nonetheless has the desirable property of smaller sampling variance and, in cases in which assessment regressivity/progressivity has not been found to be a significant concern, can provide an acceptable substitute for the weighted mean.

If samples are known to be reasonably representative through outlier trimming, the use of stratification or selection of random appraisals, the weighted mean would be the (only) correct measure. In cases which sample representativeness is a concern due to small samples or outliers, the median can reasonably be used as long as the equalization agency has checked to ensure that there are no significant price-related biases within the strata used in the study.

6.4 Measures of Variability

Measures of dispersion or variability relate to the uniformity of the ratios and should be calculated for each stratum in the study. In general, the smaller the measure, the better the uniformity, but extremely low measures can signal one of the following:

acceptable causes

- extremely homogeneous properties
- very stable markets

unacceptable causes

- lack of quality control
- calculation errors
- poor sample representativeness
- sales chasing

Note that as market activity changes or as the complexity of properties increases, the measures of variability usually increase, even though appraisal procedures may be equally valid.

6.5 Measures of Reliability

It is good practice to calculate measures of reliability whenever the results of a ratio study are used for equalization. Measures of reliability will indicate whether there is a desired degree of confidence that a given level of appraisal has not been achieved. The most commonly used measure of ratio study sample reliability is the confidence interval. This interval brackets the unknown population parameter for any sample statistic with a specified (chosen) degree of confidence. When the interval includes a desired assessment level or a performance standard range around the desired level (see section 11 and Table 2-4), equalization adjustments are not warranted. Similarly, when the interval includes a maximum allowable COD (see Table 2-3), reappraisal or other action to correct poor uniformity is not warranted.

6.6 Vertical Inequities

The measures of variability discussed in section 6.4 relate to “horizontal,” or random, dispersion among the ratios in a stratum, regardless of the value of individual parcels. Another form of inequity can be systematic differences in the appraisal of low- and high-value properties, termed “vertical” inequities. When low-value properties are appraised at greater percentages of market value than high-value properties, assessment *regressivity* is indicated. When low-value properties are appraised at smaller percentages of market value than high-value properties, assessment *progressivity* is the result. Appraisals made for tax purposes should be neither regressive nor progressive.

An index statistic for measuring vertical equity is the PRD, which is calculated by dividing the mean ratio by the weighted mean ratio. This statistic should be close to 1.00. Measures considerably above 1.00 tend to indicate assessment regressivity; measures below 1.00 suggest assessment progressivity. When samples are small or the weighted mean is heavily influenced by several extreme sales prices, however, the PRD may not be a sufficiently reliable measure of vertical inequities. A scatter plot of ratios versus appraised values or sale prices is a useful diagnostic tool. A downward (or upward) trend to the data indicates systematic regressivity (or progressivity). If not sufficiently representative, extreme sales prices can be excluded in calculation of the PRD. Similarly, when samples are very large, the PRD may be too insensitive to show small pockets in which there is significant vertical inequity. Standards for evaluating the PRD are given in section 9.2.7 in this part. In addition, more powerful statistical tests for vertical inequities are available and should be employed to determine the significance of the indication provided by the PRD (see section 5.7 in this part and Twark, Everly and Downing [1989]).

The coefficient of price-related bias (PRB) provides a more meaningful measure of price-related bias. It is obtained by regressing percentage difference from the median ratio on percentage differences in value (see Appendix D). A PRB of $-.045$ indicates, for example, that assessment ratios fall by 4.5% when values double and increase by 4.5% when values are halved. Like all regression coefficients, the statistical reliability of the PRB can be gauged by noting its *t*-value and related significance level. Like all regression coefficients, the statistical reliability of the PRB can be gauged by noting its *t*-value and related significance level, and by computing confidence intervals. In table 1-4 the PRB is 0.035 and is not statistically significant.

Unacceptable vertical inequities should be addressed through reappraisal or other corrective actions. In some cases, additional stratification can help isolate the problem. Measures of level computed for value strata should not be compared as a way of determining vertical inequity because of a boundary effect that is most pronounced in the highest and lowest strata (Schultz 1996).

6.7 Tests of Hypotheses

An appropriate test should be used whenever the purpose of a ratio study is implicitly or explicitly to test a hypothesis. A hypothesis is essentially a tentative answer to a question, such as, Are residential and commercial properties appraised at equal percentages of market value? A test is a statistical means of deciding whether the answer “yes” to such a question can be rejected at a given level of confidence. In this case, if the test leads to the conclusion that residential and commercial properties are not appraised at equal percentages of market value, some sort of corrective action on the part of assessing officials is clearly indicated. Appropriate tests are listed in table 1-2 and discussed in Gloudemans (1999), *Property Appraisal and Assessment Administration* (IAAO 1990), and *Improving Real Property Assessment* (IAAO 1978, 137–54)

6.8 The Normal Distribution

Many conventional statistical methods assume the sample data conform to the shape of a bell curve, known as the normal (or Gaussian) distribution. Performance measures based on the mean or standard deviation can be misleading if the study sample does not meet the assumption of normality. As a first step in the analysis, the distribution of sample ratios should be examined to reveal the shape of the data and uncover any unusual features. Although ratio study samples typically do not conform to the normal distribution, graphical techniques and numerical tests can be used to explore the data thoroughly. Traditional choices are the binomial, chi-square, and Lilliefors tests. Newer and more powerful procedures are the Shapiro-Wilk *W*, the D’Agostino-Pearson K^2 , and the Anderson-Darling A^2 tests (D’Agostino and Stephens 1986).

7. Sample Size

7.1 Importance of Sample Size

If it is desirable to create narrow, uniform margins of error in jurisdictions without sufficient sales, independent appraisals may be added.

7.2 Adequacy of a Given Sample Size

The adequacy of a given sample size can be evaluated by computing measures of reliability. If the confidence interval is sufficiently narrow, the sample is large enough. If the confidence interval is too wide, the oversight authority must either accept less precision or enlarge the sample, if possible.

7.3 Required Sample Size

Because designing for sampling objectives and planning for resource allocation in ratio studies must occur well before final ratio data sets are available and ratio study statistics are calculated, decisions on critical input variables must be made well before their true values are known. For example, the sample size formulas (Cochran 1977; Sherrill and Whorton 1991; and Gloude-mans 1999) used to plan for specific margins of error and/or specific levels of confidence theoretically require, as input variables, the actual variation within the final ratio data sets (usually measured by the coefficient of variation). However, the actual variation in final ratio data sets is not known during the design and planning stage and, thus, the desired sample size must be projected based upon the best information available at the time of design and planning. This projection results in unavoidable forecast error and can result in the production of a higher or lower sample size than needed to reach sampling objectives. This issue is an accepted part of conducting ratio studies when it is necessary and important to attain a predetermined or uniform degree of precision. In other cases, it may be acceptable to use all available qualified sales. When predetermination of sample size is important, the variation in the ratio data set from the most recent time period available can provide a reasonable estimate for the time period under analysis.

7.4 Remedies for Inadequate Samples

In addition to recommendations discussed in section 6.4, “Remedies for Inadequate Samples,” in Part 1, supplemental independent appraisals can be combined with sales (also see section 8.7, “Combining of Sales and Appraisals,” in this part).

7.5 History of Sales Reporting

Oversight agencies that develop ratio studies from sales provided by local assessment jurisdictions should track the number of transfers obtained in different study periods. Quality control techniques can be used to measure market activity or to determine whether an assessor is fully reporting sales information.

8. Appraisal Ratio Studies

Appraisal ratio studies are conducted by using appraised values for a random sample of parcels. Such sampling plans can be designed to be more representative of the population in terms of property characteristics than a sales sample of the same size but require adequately trained appraisers and are comparatively expensive. Few ratio studies are based solely on independently conducted appraisals, which then are compared to values determined by assessing officials. Many equalization or oversight agencies, however, do ratio studies in which both sales and appraisals are combined. Furthermore, it may be possible to develop sales driven models for use in appraising a particular population of properties (excluding those not adequately represented in the underlying model) or randomly selected parcels for ratio study purposes (see *Standard on Automated Valuation Models*, [IAAO 2003]). Estimates of value developed for use in appraisal ratio studies are considered appraisal services and must comply with *USPAP* Standards 1 and 2 or Standard 6.

8.1 Rationale

Independent appraisals can be used as indicators of market value. Independent appraisals are appraisals performed by appraisers who are not employees of the appraisal agency that is the subject of the study. Such appraisal ratio studies are particularly useful for property classes with limited sale data, such as commercial and industrial real property and personal property (see *Property Appraisal and Assessment Administration* IAAO 1990, Appendix 1-1] and Gloude-mans [1999, chapter 6]). In addition, appraisal ratio studies can be used for agricultural or other properties not appraised on an ad valorem basis. In this case, the appraisals should reflect the use value or other statutory basis on which the properties are appraised.

8.2 Advantages and Disadvantages

Appraisal ratio studies have both advantages and disadvantages. The advantages of appraisal ratio studies are

- the ability to sample from areas or property types with insufficient sales information
- a high degree of control in sample size that enables the analyst to treat jurisdictions equally, regardless of the availability of market information
- the avoidance of nonrepresentativeness stemming from the use of sales samples that may not represent the property population.
- the size of the sample can be specified and
- the initial sample can be randomly drawn, thus helping to maximize representativeness.

If objectivity can be maintained, the appraisal ratio study avoids potential distortions due to systematic differences

between appraisals of sampled and unsampled properties. In addition, independent appraisals can be used to test for systematic differences between appraisals of sold and unsold properties.

A disadvantage of appraisal ratio studies is the extra time and cost involved with the independent appraisal process. The subject and any comparables should be physically inspected and the appraisals documented according to appropriate standards. Applicable USPAP guidelines should be followed. Independent single-property appraisals should be developed in compliance with *Standard 1*, should be reported in compliance with *Standard 2*, and should be reviewed in compliance with *Standard 3* of USPAP. Independent appraisals done with a mass appraisal model should be developed and reported in compliance with *Standard 6* of USPAP. Another disadvantage is that appraisals are an opinion of value. Accordingly, they should be documented and tested against the market. However, this becomes difficult when sales data are scarce. To reduce this disadvantage, appraisal ratio study analysts should ensure that appraisals are carefully reviewed and allow local appraisers to submit appraisal information that may affect the value conclusion (see *Standard on Oversight Agency Responsibilities* [IAAO 2010]). Where adequate sales are available, independent appraisals should be checked for consistency with sales.

8.3 Sample Selection and Resource Requirements

Sample selection and resource planning in appraisal ratio studies require knowledge of statistical sampling, estimation principles, and available resources. Judgment must be used, because the determination of an adequate sample can require more information than is available during the design and planning phase, such as the actual variation within the final ratio data sets (see section 6.2, “Adequacy of a Given Sample Size,” in Part 1). Moreover, the cost of the study increases with the size of the sample. Therefore, the value of more reliable information must be balanced against the costs of obtaining that information.

In determining the size of the sample for each stratum, the following should be taken into consideration:

1. the required precision (typically measured by the margin of error) of the estimate of the appraisal level, for example, ± 0.05
2. the required confidence level, for example, 95 percent
3. the amount of dispersion expected in the final ratio data set
4. the wastage associated with properties that cannot be efficiently appraised or appraisals that cannot be used for one reason or another (see Gloude-mans [1999, chapter 6] for sample size

formulas and required input variables; also see Sherrill and Whorton [1991]).

Once the desired size of an appraisal sample has been determined, the individual properties that will constitute the sample should be selected using a statistically valid sampling plan. Stratified random sampling is preferred.

If value stratification is used, sample properties selected from value groups during resource planning can shift into other value groups before completion of the study, thus reducing the ultimate representativeness of the sample. Some appraisal parcels may need to be removed from the sample when anomalous conditions are discovered such as environmental contamination (sufficiently reliable valuations may be prohibitively difficult or resource intensive) or when the independent appraiser is not allowed access to the property. Any sample parcels that are voided or that shift from a stratum because of value changes should be replaced if possible.

Appraisal ratio studies, as with sales ratio studies, require informed, reasoned judgment to maximize sample representativeness and statistical reliability.

8.4 Data Requirements and Appraisal Techniques

The appraisal techniques selected for an appraisal ratio study should be consistent with accepted appraisal principles and practices. The appraisals should reflect the appraisal date in question and should be well documented. Statistical software should be used as much as possible to expand analytical capabilities and perform calculations.

The appraisals used in appraisal ratio studies can be based on CAMA and automated valuation model (AVM) techniques (see *Standard on Automated Valuation Models*, [IAAO 2003]). The models used must be developed independently from those used for assessment purposes. Adequate market data and property characteristic data are required to develop reliable and defensible model estimates. If available, sales from a later period can be used to expand sample size. However, as in sales-based ratio studies, sales derived from primary assessing jurisdictions should be reviewed to ensure accuracy and validity. CAMA and AVM models have the advantage of reducing costs, permitting the use of larger, more representative samples. CAMA and AVM models developed for equalization must focus on the adequacy of overall, not individual, value or level of assessment estimates.

Because the purpose of the appraisal is to make an *independent* value estimate, not audit the assessor’s work, the appraisals should be made without knowledge of the assessor’s value. Appraisers should *not* be supplied with copies of the assessor’s appraisal work sheets or model information. Supervisors should spot-check and review the work of staff appraisers to ensure that the required independence is maintained. When the purpose of the ratio study is equal-

ization or performance measurement, rather than internal quality assurance, the appraisals should not be revealed to the assessor until the assessor's values are final.

8.5 Appraisal Chasing

Appraisal chasing can take two forms, either of which reduces or destroys the validity of the ratio study. The first occurs when an independent appraiser knows the local appraised value and either consciously or unconsciously biases the independent appraised value towards the local appraised value. Independent appraisers should not have access to the local appraiser's values or appraisal work papers prior to completing their appraisals. Also, independent appraisals should be reviewed and tested against the market.

The second form of appraisal chasing occurs when the local appraisal jurisdiction knows which properties are in the ratio study appraisal sample and adjusts local appraised values on some or all of these properties to achieve better ratios without making similar adjustments to unsampled properties. This form of appraisal chasing is similar to sales chasing and has similar consequences (see Appendix E, "Sales Chasing Detection Techniques"). Ratio study analysts should guard against this form of appraisal chasing by withholding the release of sample information until the local appraisal office's values are final. If this form of appraisal chasing occurs, the oversight agency can use local values prior to adjustment to provide a more accurate representation of the population ratios.

8.6 Reviewing of Appraisals

Appraisal supervisors should review appraisal models or individual single-property appraisals to ensure that USPAP and the agency's standards are met. It also is good practice to include some recently sold properties in the sample being appraised as a check on the validity of the methods being applied. In addition, the assessor must be afforded an opportunity to review the appraisals along with supporting documentation and to submit information supporting different value conclusions. If different value conclusions or factual information would materially affect the outcome of the study, a procedure for resolving conflicts, for example, by an independent review body, should be established.

8.7 Combining of Sales and Appraisals

Appraisals can be combined with valid sales in a ratio study. Using available sales adds objectivity to the study and reduces the required number of appraisals. On the other hand, combining sales and appraisals mixes two market indicators. If sales and appraisals are combined, an analysis should be performed to test the consistency of measures of central tendency derived from the sales ratios compared to the same measures derived from the appraisal ratios. A Mann-Whitney test comparing values per unit or comparing ratios based on sales with those based on appraisals is

appropriate for this purpose. Significant differences can result from several of the following conditions:

1. Sales have been chased.
2. Sales and appraisals came from different geographic areas with different markets and different levels of appraisal (maximize representativeness by stratifying).
3. Sales and appraisals have different property characteristics that cause different levels of appraisal.
4. All or some of the sales are invalid.
5. Outlier ratios are causing sale/appraisal ratio differences.
6. All or some of the appraisals are inaccurate.

If none of the first five conditions listed above apply, the appraisals should be tested against the market and revised as necessary (see Wooten, 2003).

Variability measures computed on sales used in the sample should not be expected to be similar to variability measures computed on appraisals. Sales ratios reflect the vagaries of the marketplace. Appraisal ratios, on the other hand, come from comparing the results of one appraisal model (the oversight agency's) to the results of another (the assessing office's). If both parties use mass appraisal procedures, differences in appraisals between the two models should be less than when compared with sales; thus, variability measures based on appraisal ratios can be expected to be lower than those based on sales ratios as long as they represent properties with similar characteristics and similar degrees of appraisal difficulty.

8.8 Average Unit Value Comparisons

In addition to a traditional ratio study, "expert" appraisals can take the form of average unit values and be compared against the assessor's average unit value for the same parcels. In this technique, parcels are stratified into homogeneous groups, as they would be for appraisal purposes. Appropriate units of comparison are identified for each group, and average unit values are determined through an analysis of available sales, cost, and income data. The assessor's average unit values for the same strata are then calculated and the two averages are compared. Average unit value comparisons is well-rooted in mass appraisal theory and offers an alternative to the time and expense associated with the selection and appraisal of individual parcels.

9. Estimating Performance for Unsold Properties

The objective of a ratio study is to determine appraisal performance for the population of properties. As long as sold and unsold parcels are appraised in the same man-

ner and the data describing them are coded consistently, statistics calculated in a sales ratio study can be used to infer appraisal performance for unsold parcels. However, if parcels that sell are selectively reappraised or recoded, based on their sale prices or some other criterion (such as listing price) and if such parcels are in the ratio study, sales ratio study uniformity inferences will not be accurate (appraisals will appear more uniform than they are). In this situation, measures of appraisal level will also be unsupported unless similar unsold parcels were appraised by a model that produces the same overall percentage of market value (appraisal level) as the parcels that sold.

Oversight agencies must ensure that sold and unsold parcels are appraised at the same level. Several techniques are available for determining whether assessors are selectively appraising sold parcels (see Appendix E, “Sales Chasing Detection Techniques,” or *Property Appraisal and Assessment Administration* [IAAO 1990, Appendix 20-2] and Gloudemans [1999, chapter 6] for a more detailed discussion).

If unsold properties within a properly specified group are not appraised consistently with sold properties within the same group and according to applicable guidelines, unadjusted sales ratio results cannot be used. The oversight agency will have to adjust calculated results or conduct an alternative study.

Once it is determined that *sales chasing* probably has occurred and probably is reducing the validity of ratio study statistical measures of level or uniformity, it is necessary to redo the ratio study to establish valid measures before any other recommendations, such as reappraisal or equalization action, can be made. If feasible, probably the best approach is to select a sample period that effectively precludes sales chasing. For example, when the lien or appraisal date is January 1, many jurisdictions use sales occurring before that date to make valuation decisions. To test the resulting valuations, it would be appropriate to use sales occurring after January 1 (or after the last date for changing assessments for the year in question), provided such data are time-adjusted (when necessary) backward to match the appraisal date. As a slight variation on this principle, earlier sales could be used, except when sales chasing is detected, in which case it is appropriate to switch to a later, post-appraisal-date sales period.

Legal or practical constraints can prevent use of optimal sample periods in many cases. In these situations, it is important to determine the exact cause of the sales chasing. For example, if a large proportion of selling properties are appealed and if appeal boards typically adjust to sale price, the result is the same as sales chasing by the assessor. One solution is to use appraised values prior to the action of the appeal board, provided that the appeal adjustment is not merely the result of an atypical clerical or other error. Another approach is to use current sales prices and prior-year values, adjusted for reappraisal

activity or assessment value changes in the population. The percentage increase or decrease in the prior-year’s appraised values for the population (net of new construction) should be used to adjust the prior-year’s values for the sample (Gloudemans 1999).

10. Presentation of Findings, Documentation, and Training

Oversight agencies should produce ratio studies in a manner that is transparent in all stages to all stakeholders.

(See section 8, Part 1.)

11. Ratio Study Standards

Each state and province should have ratio study performance standards. These standards, summarized in table 2-3, are suggested for jurisdictions in which current market value is the legal basis for assessment. In general, when state and provincial standards are not met, reappraisal or other corrective measures should be taken or equalization procedures can be imposed. When an oversight agency orders such actions, the burden of proof should be on the agency to show that the standards have not been achieved.

All standards recommended in this section are predicated on the assumption that all practicable steps necessary to maximize representativeness and validity in the underlying ratio studies have been conducted.

11.1 Level of Appraisal

The calculated measures of central tendency are point estimates and provide only an indication, not proof, of whether the level meets the appropriate goal. Confidence intervals and statistical tests should be used to determine whether the appraisal level differs from the established goal in a particular instance.

A decision by an oversight agency to take some action (direct equalization, indirect equalization, reappraisal) can have profound consequences for taxpayers, taxing jurisdictions, and other affected parties. This decision should not be made without a high degree of certainty that the action is warranted. Conversely, a decision not to take action when action is needed can have equally profound consequences. Oversight agencies should weigh all the options and consider the issues discussed below when developing or revising a level-of-appraisal standard, and when developing equalization or other appraisal oversight procedures.

11.1.1 Purpose of Level-of-Appraisal Standard

Jurisdictions that follow the IAAO recommendation of annual reassessments and comply with USPAP standards should be able to develop mass appraisal models that maintain an overall ratio level of 100 percent (or very near thereto). The local assessor may be required to observe reap-

Table 2-3. Ratio study uniformity standards indicating acceptable general quality*

General Property Class	Jurisdiction Size/Profile/Market Activity	COD Range
Residential improved (single family dwellings, condominiums, manuf. housing, 2-4 family units)	Very large jurisdictions/densely populated/newer properties/active markets	5.0 to 10.0
	Large to mid-sized jurisdictions/older & newer properties/less active markets	5.0 to 15.0
	Rural or small jurisdictions/older properties/depressed market areas	5.0 to 20.0
Income-producing properties (commercial, industrial, apartments,)	Very large jurisdictions/densely populated/newer properties/active markets	5.0 to 15.0
	Large to mid-sized jurisdictions/older & newer properties/less active markets	5.0 to 20.0
	Rural or small jurisdictions/older properties/depressed market areas	5.0 to 25.0
Residential vacant land	Very large jurisdictions/rapid development/active markets	5.0 to 15.0
	Large to mid-sized jurisdictions/slower development/less active markets	5.0 to 20.0
	Rural or small jurisdictions/little development/depressed markets	5.0 to 25.0
Other (non-agricultural) vacant land	Very large jurisdictions/rapid development/active markets	5.0 to 20.0
	Large to mid-sized jurisdictions/slower development/less active markets	5.0 to 25.0
	Rural or small jurisdictions/little development/depressed markets	5.0 to 30.0

These types of property are provided for general guidance only and may not represent jurisdictional requirements.

**The COD performance recommendations are based upon representative and adequate sample sizes, with outliers trimmed and a 95% level of confidence.*

**Appraisal level recommendation for each type of property shown should be between 0.90 and 1.10.*

**PRD's for each type of property should be between 0.98 and 1.03 to demonstrate vertical equity. However, PRD standards are not absolute and may be less meaningful when samples are small or when wide variation in prices exist. In such cases, statistical tests of vertical equity hypotheses should be substituted.*

**Alternatively, assessing officials can rely on the PRB, which is less sensitive to atypical prices and ratios. PRB coefficients should generally fall between $-.05$ and $.05$. PRBs that are statistically significant and less than -0.10 or greater than 0.10 indicate unacceptable vertical inequities.*

**CODs lower than 5.0 may indicate sales chasing or non-representative samples.*

praisal cycles defined by a legal authority or public policy that can extend beyond one year. During extended cycles inflation or deflation can influence the overall ratio.

The purpose of a performance standard that allows reasonable variation from 100 percent of market value is to recognize uncontrollable sampling error and the limiting conditions that may constrain the degree of accuracy that is possible and cost-effective within an assessment jurisdiction. Further, the effect of performance standards on local assessors must be considered in light of expectations of public policy and resources available. For these reasons, states or oversight agencies may adopt performance standards for appraisal level that allow some variance from the 100 percent goal of market value.

11.1.2 Recommended Appraisal Level Standards for Direct and Indirect Equalization

The performance standard adopted by an oversight agency should be a range around the legally required level of appraisal in a property class or an overall jurisdiction. This range should be 90 to 110 percent of the legally required level of appraisal for direct equalization or reappraisal, or 95 to 105 percent for indirect equalization. A smaller maximum range for indirect equalization is justified because taxpayers are not as comprehensively affected. Oversight agencies should adopt performance standards that are as close to the legally required level as can be justified given the local situation and taking into account

the factors discussed herein.

In addition to the above appraisal level standards, each class of property for which appraisal level standards have been defined must be within 5 percent of the overall level of appraisal of the jurisdiction (see section 11.2.3, “Uniformity among Strata,” in this part). Both criteria must be met.

11.1.3 Confidence Intervals in Conjunction with Performance Standards

By themselves, the calculated measures of central tendency provide only an indication, not proof, of whether the appraisal level meets the performance standard. So, the purpose of confidence intervals and similar statistical tests is to determine whether the appraisal level differs from the established performance standard in a particular instance. A conclusion of noncompliance requires a high degree of confidence, thus a 90 percent (two-tailed) or 95 percent (one-tailed) confidence interval should be used, except for small or highly variable samples as described in section 11.1.5, “Adjustment for High Variability and Small Samples,” in this part.

11.1.4 Decision Model

The oversight agency should determine whether the estimate is outside the acceptable range around the legal level of appraisal with a specified degree of statistical significance. The chosen interval should overlap the performance standard range of 90 percent to 110 percent

in the case of direct equalization or measuring appraisal performance. For indirect equalization the chosen interval should overlap the performance standard range of 95 percent to 105 percent. If the confidence interval does not overlap any portion of the appropriate range, equalization is performed or reappraisal orders are issued. See table 2-4 for an example of the direct equalization or appraisal performance decision making process.

11.1.5 Adjustments for High Variability and Small Samples

High variability, small sample size, or a combination of these factors often causes confidence intervals to become quite wide. Wide confidence intervals reflect the imprecision of the underlying statistic and can decrease the usefulness of performance measures. Also, wide confidence intervals can cause an inequitable situation in which jurisdictions with small samples and large variability are never subject to equalization or reappraisal orders, while jurisdictions with larger samples and much less variability are more likely to be subject to such orders even though their appraisal performance may be arguably better.

For these reasons, oversight agencies should consider expanding sample sizes by taking steps to increase the number of sales or by making independent appraisals (see section 7.4 part 2). If the sample size cannot be increased, two options may be considered when the point estimate fails to achieve compliance but the confidence interval overlaps the range of compliance:

- If a particular point estimate does not meet the standard for the current study cycle the oversight agency may reduce the level of confidence by 5% the following year. This may be followed by an annual stepwise reduction of 5%. Such a reduction may continue to a 70 percent level of confidence if the point estimate fails to meet the compliance threshold over this period of time. Corrective action would be imposed when a given year's confidence interval fails to include the performance standard range.
- The oversight agency may examine statistical point estimates over several study cycles. A jurisdiction that fails to meet a particular point standard for 5 consecutive years has a probability of less than 5% that compliance has been achieved, even if

the confidence interval overlaps the compliance threshold every year. In such cases the oversight agency would impose corrective decisions based upon the point estimate.

11.1.6 Calculating Equalization Adjustments

If noncompliance with either direct or indirect equalization standards is indicated, the appropriate point estimate (statistic) measuring appraisal level should be used to calculate adjustment factors, by dividing it into 100 percent.

11.2 Appraisal Uniformity

Assuming the existence of an adequate and sufficiently representative sample, if the uniformity of appraisal is unacceptable, reappraisal should be undertaken regardless of the level of appraisal. The oversight agency should recognize that the COD is a point estimate and cannot be accepted as proof of assessment uniformity problems without an appropriate degree of statistical confidence. Such proof can be provided by recognized statistical tests, including bootstrap confidence intervals. If the data are normally distributed, the COV and confidence intervals around this measure also can be determined. Then the COV can be mathematically converted into an equivalent COD.

11.2.1 Oversight Uniformity Standards

Oversight agencies should establish uniformity standards for local assessment jurisdictions. Any COD performance standards applied to strata within a particular jurisdiction should be related to the overall size, profile of property characteristics (type, age, condition, and obsolescence) and market activity. In general, tighter uniformity standards can be applied to larger jurisdictions with newer construction and active markets. And generally, less stringent uniformity standards should be applied to older, economically depressed or less densely developed areas with less efficient markets. Standards should also be relaxed in jurisdictions that experience economic instability due to sudden changes in supply or demand factors. In developing uniformity standards, oversight agencies should consider reasonable tolerance ranges in making compliance decisions.

11.2.2 Multi-level Uniformity Standards

The uniformity standards presented in table 2-3 are defined in terms of the COD (point estimate) measure and are

Table 2-4. Ratio Study Standards and Decision Making—Direct Equalization or Appraisal Performance Using Median 90%–110% Standard

Example demonstrating application of standard at a 95% level of confidence

Case	Point Estimate	Confidence Interval (CI) Width (95%)	CI Overlaps Performance Standard Range	Point Estimate in Performance Standard Range	Equalization Action or Reappraisal Order
1	92%	86% to 101%	yes	yes	no
2	88%	81% to 95%	yes	no	no
3	84%	79% to 88%	no	no	yes

intended to apply to ratio studies based on sales, not those based on independent appraisals in which lower CODs often are typically observed. If reliability measures are not employed, sample size will play a critical role in setting the maximum acceptable COD. In addition, in unusually homogeneous or restrictive markets or for properties subject to use-value or similar constrained value assessment, low CODs also can be anticipated. In all other cases, CODs less than 5 percent should be considered unusual and possibly indicative of nonrepresentative samples or the selective reappraisal of sold parcels. The COD standards in table 2-3 may not be applicable to property strata in unique, depressed, or rapidly changing markets. In such cases, assessment administrators may be able to develop target standards based on an analysis of past performance or results in similar markets elsewhere. Such an analysis can be based on ratio study results for the past five years or more.

11.2.3 Uniformity among Strata

Although the goal is to achieve an overall level of appraisal equal to 100 percent of the legal requirement, ensuring uniformity in appraisal levels among strata is also important. The level of appraisal of each stratum (class, neighborhood, age group, market areas, and the like) should be within 5 percent of the overall level of appraisal of the jurisdiction. For example, if the overall level of appraisal of the jurisdiction is 1.00, but the appraisal level for residential property is 0.93 and the appraisal level for commercial property is 1.06 the jurisdiction is not in compliance with this requirement. This test should be applied only to strata subject to compliance testing. The oversight agency can conclude that this standard has been met if 95 percent (two-tailed) confidence intervals about the chosen measures of central tendency for each of the stratum fall within 5 percent of the overall level of appraisal calculated for the jurisdiction. Using the above example, if the upper confidence limit for the level of residential property is 0.97 and the lower confidence limit for commercial property is 1.01, the two strata are within the acceptable range.

11.2.4 Vertical Equity

PRDs should be between 0.98 and 1.03. The reason this range is not centered on 1.00 relates to an inherent upward bias in the arithmetic mean (numerator in the PRD) that does not equally affect the weighted mean (denominator in the PRD). When samples are small, have high dispersion, or include properties with extreme values, the PRD may not provide an accurate indication of assessment regressivity or progressivity. When relying on the PRD to measure vertical equity, it is good practice to perform an appropriate statistical test for price-related biases before concluding that they exist (see table 1-2 in Part 1).

The PRB provides a measure of price-related bias that is more meaningful and less sensitive to extreme prices or ratios. As a general matter, the PRB coefficient should fall between -0.05 and 0.05 . PRBs for which 95% confidence intervals fall outside of this range indicate that one can reasonably conclude that assessment levels change by more than 5% when values are halved or doubled. PRBs for which 95% confidence intervals fall outside the range of -0.10 to 0.10 indicate unacceptable vertical inequities.

As an illustration of the above, assume that the PRB is -0.115 with a standard error of 0.02 and corresponding 95% confidence interval of -0.075 to -0.155 (-0.115 ± 0.04 approximately). One can conclude with 95% confidence that assessment levels change by at least 7.5% when values double or are halved but not that assessment levels change by at least 10%. This result would not be out of compliance with the ± 0.10 standard.

11.3 Natural Disasters and Ratio Study Standards

Natural disasters such as earthquakes, floods, and hurricanes can have a substantial impact on the conduct of ratio studies and the interpretation and use of the results, and in general, they:

- increase the difficulty of accurately identifying the physical and economic characteristics of property on the dates of sale/lease and the date of appraisal
- increase the difficulty of producing sufficiently reliable appraised values (numerators)
- decrease the availability of usable sales and other market data
- increase the difficulty of identifying and obtaining such usable data
- increase the difficulty of producing sufficiently reliable independent appraisals
- increase the difficulty of accurately matching the characteristics of numerators with those of denominators

These potential problems can result from extraordinary changes in market conditions and in the physical and economic characteristics of property between the dates of sale/lease and the date of appraisal. As a result of these potential problems, a number of unreliable sample properties may need to be voided and usable sample sizes can be reduced significantly. All of these factors should be considered when ratio study standards are applied to ratio study results from areas substantially affected by natural disasters, but such consideration must not result in unwarranted relaxation of applicable standards. When faced with such situations, oversight agencies must use informed, reasoned judgment and common sense to pro-

duce a sufficiently reliable ratio study, based upon the best information available.

12. Personal Property Studies

Most personal property ratio studies performed by oversight agencies are performed for equalization purposes. Because indirect equalization in particular requires overall estimation of value, it is imperative for these ratio studies to focus on large accounts.

Horizontal equity requires similar levels of appraisal between real and personal property. Sales data for personal property are difficult to obtain and analyze because markets for personal property are generally less visible and more difficult to follow than real property markets. Therefore, performance reviews and appraisal ratio studies should be used in place of sales ratio studies to determine the quality of appraisal of personal property. The performance review does not quantify assessment conditions but can determine general assessment quality. The appraisal ratio study can be used to determine the level and uniformity of assessment for personal property.

12.1. The Performance Review

The performance review is an empirical study that evaluates the assessment method used and the ability of the jurisdiction to meet its legal requirement in the assessment of personal property. This type of study can be used to allocate tax dollars in multijurisdictional funding calculations or equalization by assuming that jurisdictions passing the performance review are assessing personal property at the general level of other classes of property analyzed with ratio studies.

12.1.1. Discovery

The jurisdiction must have the ability to discover the owners or users of taxable personal property within the jurisdiction. This is accomplished using phone books, business/occupational licenses, listings, sales tax rolls, and field reviews (see IAAO Course 500, “The Assessment of Personal Property,” and *Standard on Valuation of Personal Property* [IAAO 2005] for a complete list).

12.1.2. Valuation

Personal property is valued by using acceptable schedules and methods including depreciation schedules published by nationally recognized valuation firms, market data from published valuation guides, and other generally accepted valuation methods and acceptable adjustments (see *Standard on Valuation of Personal Property*).

12.1.3. Verification

Inclusiveness of personal property returns and reports should be verified by an audit program. The audit program should focus on larger and complex accounts; however, it also should include randomly selected accounts. The audit program should provide coverage of the entire tax base regardless of the jurisdiction’s reappraisal cycle.

12.1.4 Forms and Renditions

Comprehensive forms supplied by the assessment authority should allow the taxpayer to disclose fully all assessable personal property. The tax laws should require mandatory compliance, with meaningful penalties for noncompliance.

12.2. Appraisal Ratio Studies for Personal Property

The appraisal ratio study produces an estimate of the level of assessment of personal property by developing a ratio for property that is on the tax roll through the use of appraisals. The level of assessment determined in this way can be adjusted downward to account for property that has not been assessed.

12.2.1 Assessment Ratio for Personal Property

Personal property market values are usually derived from appraisals using a replacement cost new less depreciation (RCNLD) approach (see IAAO Course 500). A comparison of the depreciation schedules in use to nationally accepted schedules would enable the calculation of a ratio for property on the roll. A statistically sound process should be used to select a sample that is representative of personal property on the tax rolls. Such a sample can be parcel- or value-based depending on the intended use of the ratio study in indirect or direct equalization.

12.2.2 Stratification

Proper stratification of personal property accounts should be done for greater statistical accuracy. Strata should be based on the type and value of personal property accounts.

Stratification by type of account should occur first. Personal property accounts can be divided into residential (motor vehicles, boats, aircraft, and the like), agriculture, and business accounts. Further stratification can occur in residential and agricultural accounts but is necessary in business or commercial accounts. Business accounts are usually stratified by size into a minimum of four groups. Value ranges for these groups should be derived from the value ranges in the local market. One example would be small (less than \$250,000), medium (\$250,000 to \$1 million), moderate (\$1–\$5 million), and large (greater than

\$5 million). Individual size of account can be determined by value on the prior-year personal property roll.

12.2.3 Property Escaping Assessment

Personal property is particularly prone to escaping assessment. Some determination should be made about the portion of taxable personal property not on the assessment roll. However, estimates based on national averages are less meaningful at the local jurisdictional level.

12.2.3.1 Identifying Personal Property Owners and Users Not on the Roll

Discovery tools can be used to determine accounts not on the roll for a sample area or group. Once the extent of the problem is identified, a projection can be made of the percentage of personal property not identified on the assessment roll.

12.2.3.2 Identifying Personal Property Not Included in Taxpayer Returns/Reports

The accepted method of determining the property omitted in taxpayer returns/reports is to audit the account

(see IAAO workshops on auditing). The audit results are applied back to the account value. The resulting fraction is property that is escaping taxation within that particular personal property account. If appropriate sampling techniques are used in selecting the accounts for audit, the resulting ratio is applied to the total roll to help determine the percentage of personal property escaping assessment within the jurisdiction.

12.2.4 Computing the Level of Appraisal

The overall ratio is then determined by reducing the valuation ratio by the percent of property wholly or partially escaping taxation. For example, if the appraisal level is found to be 90 percent and it is determined that 5 percent of personal property is escaping assessment, then the corrected level of assessment is the appraisal level times the percentage of personal property assessed: $0.90 \times (1 - 0.05) = 0.855$. For indirect equalization, this calculation would result in a higher equalized value.

Standard on Ratio Studies

Definitions

Absolute value. The value of a number (or variable) regardless of its sign. For example, 3 and -3 (minus 3) both have an absolute value of 3. The mathematical symbol for absolute value is one vertical bar on each side of the number in question, for example, $|3|$.

Accuracy. The closeness of a measurement, computation, or estimate to the true, exact, or accepted value. Accuracy also can be expressed as a range about the true value. *See also precision and statistical accuracy.*

Adjusted sale price. The sale price that results from adjustments made to the stated sale price to account for the effects of time, personal property, financing, or the like.

Appraisal. “The act or process of developing an opinion of value; an opinion of value” (USPAP 1999). The act of estimating the money value of property. The money value of property as estimated by an appraiser.

Appraisal date. The date as of which a property’s value is estimated. *See also assessment date.*

Appraisal ratio. (1) The ratio of the appraised value to an indicator of market value. (2) By extension, an estimated fractional relationship between the appraisals and market values of a group of properties. *See also level of appraisal.*

Appraisal ratio study. A ratio study using independent expert appraisals as indicators of market value.

Appraisal-sale price ratio. The ratio of the appraised value to the sale price (or adjusted sale price) of a property; a simple indication of appraisal accuracy.

Appraised value. The estimate of the value of a property before application of any fractional assessment ratio, partial exemption, or other adjustments.

Arithmetic mean. A measure of central tendency. The result of adding all the values of a variable and dividing by the number of values. For example, the arithmetic mean of 3, 5, and 10 is 18 divided by 3 or 6.

Array. An ordered arrangement of data, such as a listing of sales ratios, in order of magnitude.

Assessed value. (1) A value set on real estate and personal property by a government as a basis for levying taxes. (2) The monetary amount at which a property is put on the assessment roll for purposes of computing the tax levy. Assessed values differ from the assessor’s estimate of actual (market) value for four major reasons: fractional assessment ratios, partial exemptions, preferential assessments, and decisions by assessing officials to override market value.

Assessment. (1) In general, the official acts of determining the amount of the tax base. (2) As applied to property taxes, the official act of discovering, listing, and appraising property, whether performed by an assessor, a board of review, or a court. (3) The value placed on property in the course of such act.

Assessment-appraisal ratio. The ratio of the assessed value of a property to an independent appraisal.

Assessment date. The status date for tax purposes. Appraised values reflect the status of the property and any partially completed construction as of this date.

Assessment progressivity (regressivity). An appraisal bias such that high-value properties are appraised higher (or lower) than low-value properties in relation to market values. *See also price-related differential (PRD) and coefficient of price-related bias (PRB).*

Assessment ratio. (1) The fractional relationship of an assessed value to the market value of the property in question. (2) By extension, the fractional relationship of the total of the assessment roll to the total market value of all taxable property in a jurisdiction. *See also level of assessment.*

Assessment-sale price ratio. The ratio of the assessed value to the sale price (or adjusted sale price) of a property.

Assessor. (1) The head of an assessment jurisdiction. Assessors can be either elected or appointed. In this standard the term is sometimes used collectively to refer to all assessment officials charged with administering the assessment function. (2) The public officer or member of a public body whose duty it is to make the original assessment.

Average deviation. The arithmetic mean of the absolute deviations of a set of numbers from a measure of central tendency such as the median. Taking absolute values is generally understood without being stated. The average deviation of the numbers 4, 6, and 10 about their median (6) is $(2 + 0 + 4) \div 3 = 2$. The average deviation is used in computing the coefficient of dispersion (COD).

Bias. A type of nonsampling error in which a calculated statistic differs systematically from the population parameter. A process is biased if it produces results that vary systematically with some factor that should be irrelevant. In assessment administration, assessment progressivity (regressivity) is one kind of possible bias.

Bootstrap. A computer-intensive method of statistical inference that is based on a repeated resampling of data to provide more information about the population charac-

teristics. The bootstrap is a data-driven procedure that is particularly useful for confidence interval approximation when no traditional formulas are available or the sample has been drawn from a population that does not conform to the normal distribution.

CAMA. See **computer-assisted mass appraisal**

Central tendency. (1) The tendency of most kinds of data to cluster around some typical or central value, such as the mean or median. (2) By extension, any or all such statistics. Some kinds of data, however, such as the weights of cars and trucks, may cluster about two or more values, and in such circumstances the meaning of central tendency becomes unclear. This may happen in ratio studies in which two or more classes of property are combined.

Class. A set of items defined by common characteristics. (1) In property taxation, property classes such as residential, agricultural, and industrial may be defined. (2) In assessment, building classification systems based on type of building design, quality of construction, or structural type are common. (3) In statistics, a predefined category into which data may be put for further analysis. For example, ratios may be grouped into the following classes: less than 0.500, 0.500 to 0.599, 0.600 to 0.699, and so forth.

COD. See **coefficient of dispersion.**

Coefficient of concentration. The percentage of observations falling within a specified percentage (say, 15 percent) of a measure of central tendency.

Coefficient of dispersion (COD). The average deviation of a group of numbers from the median expressed as a percentage of the median. In ratio studies, the average percentage deviation from the median ratio.

Coefficient of price-related bias (PRB). An index of price-related bias obtained by regressing percentage deviations from the median ratio on percentage changes in a value proxy, which is obtained by giving equal weight to assessments and sales prices so as to minimize measurement biases.

Coefficient of variation (COV). A standard statistical measure of the relative dispersion of the sample data about the mean of the data; the standard deviation expressed as a percentage of the mean.

Computer-assisted mass appraisal (CAMA). A process that uses a system of integrated components and software tools necessary to support the appraisal of a universe of properties through the use of mathematical models that represent the relationship between property values and supply/demand factors.

Confidence interval. A range of values, calculated from the sample observations, that are believed, with a particular probability, to contain the true population parameter (mean, median, COD). The confidence interval is not

a measure of precision for the sample statistic or point estimate, but a measure of the precision of the sampling process (see **reliability**).

Confidence level. The degree of probability associated with a statistical test or confidence interval, commonly 90, 95, or 99 percent. For example, a 95 percent confidence interval implies that were the estimation process repeated again and again, then 95 percent of the calculated intervals would be expected to contain the true population measure (such as the median, mean, or COD).

Contributory value. The amount a component of a property contributes to the total market value. For improvements, contributory value must be distinguished from costs.

COV. See **coefficient of variation.**

Date of sale (date of transfer). The date on which the sale was consummated. This is considered to be the date the deed, or other instrument of transfer, is signed. The date of recording can be used as a proxy if it is not unduly delayed as it would be in a land contract.

Direct equalization. The process of converting ratio study results into adjustment factors (trends) and changing locally determined appraised or assessed values to more nearly reflect market value or the legally required level of assessment. See also **equalization** and **indirect equalization**.

Dispersion. The degree to which data are distributed either tightly or loosely around a measure of central tendency. Measures of dispersion include the range, average deviation, standard deviation, coefficient of dispersion, and coefficient of variation.

Distribution-free statistics. A set of robust nonparametric methods whose interpretation or reliability does not depend on stringent assumptions about the distribution of the underlying population from which the sample has been drawn. See also **parametric statistics**.

Equalization. The process by which an appropriate governmental body attempts to ensure that property under its jurisdiction is assessed at the same assessment ratio or at the ratio or ratios required by law. Equalization can be undertaken at many different levels. Equalization among use classes (such as agricultural and industrial property) can be undertaken at the local level, among properties in a school district and a transportation district; equalization among counties is usually undertaken by the state to ensure that its aid payments are distributed fairly. See also **direct equalization** and **indirect equalization**.

Exploratory data analysis. That part of statistical practice concerned with reviewing the data set to isolate structures, uncover patterns, or reveal features that may improve the confirmatory analysis.

Fixture. An asset that has become part of real estate through attachment in such a manner that its removal

would result in a loss in value to either the asset or the real estate to which the asset is affixed.

Fractional assessments. Assessments that by law or by practice have assessment ratios different from 1. Usually the assessment ratio is less than 1, and if assessment biases are present, different classes of property may have different fractional ratios.

Frequency distribution. A table or chart showing the number or percentage of observations falling in the boundaries of a given set of classes. Used in ratio studies to summarize the distribution of the individual ratios. *See also class and histogram.*

Histogram. A bar chart or graph of a frequency distribution in which the frequencies of the various classes are indicated by horizontal or vertical bars whose lengths are proportional to the number or percentage of observations in each class.

Hypothesis. A statement in inferential statistics, the truth of which the analyst is interested in determining.

Independent appraisal. An estimate of value using a model different from that used for assessment purposes. Independent appraisals are used to supplement sales in sales ratio studies or in appraisal ratio studies.

Indirect equalization. The process of computing hypothetical values that represent the oversight agency's best estimate of taxable value, given the legally required level of assessment or market value. Indirect equalization allows proper distribution of intergovernmental transfer payments between state or provincial and local governments despite different levels of appraisal between jurisdictions or property classes. *See also equalization and direct equalization.*

Interquartile range (IQR). The result obtained by subtracting the first quartile from the third quartile. By definition 50 percent of the observations fall within the IQR.

Land contract. An executor's contract for the purchase of real property under the terms of which legal title to the property is retained by the vendor until such time as all conditions stated in the contract have been fulfilled; commonly used for installment purchase of real property.

Level of appraisal. The common, or overall, ratio of appraised values to market values. Three concepts are usually of interest: the level required by law, the true or actual level, and the computed level based on a ratio study.

Level of assessment. The common or overall ratio of assessed values to market values. *See also level of appraisal.* *Note:* The two terms are sometimes distinguished, but there is no convention determining their meanings when they are. Three concepts are commonly of interest: what the assessment ratio is legally required to be, what the assessment ratio for the population actually is, and what

the assessment ratio for the population seems to be, on the basis of a sample and application of inferential statistics. When level of assessment is distinguished from assessment ratio, *level of assessment* usually means either the legal requirement or the true ratio, and *assessment ratio* usually means the true ratio or the sample statistic.

Margin of error. A measure of the uncertainty associated with statistical estimates of a parameter. It is typically linked to consumer surveys or political poll questions. A margin of error is a key component of a confidence interval. It reports a "plus or minus" percentage or proportion quantity in a confidence interval at a specified level of probability (typically 95 percent). *See also confidence interval.*

Market value. The major focus of most real property appraisal assignments. Both economic and legal definitions of market value have been developed and refined. A current economic definition agreed upon by agencies that regulate federal financial institutions in the United States is: The most probable price (in terms of money) which a property should bring in a competitive and open market under all conditions requisite to a fair sale, the buyer and seller each acting prudently and knowledgeably, and assuming the price is not affected by undue stimulus. Implicit in this definition is the consummation of a sale as of a specified date and the passing of title from seller to buyer under conditions whereby: The buyer and seller are typically motivated; Both parties are well informed or well advised, and acting in what they consider their best interests; A reasonable time is allowed for exposure in the open market; Payment is made in terms of cash in United States dollars or in terms of financial arrangements comparable thereto; The price represents the normal consideration for the property sold unaffected by special or creative financing or sales concessions granted by anyone associated with the sale. (See USPAP for additional comments.)

Mass appraisal. The process of valuing a universe of properties as of a given date using standard methodology, employing common data, and allowing for statistical testing (see *USPAP*)

Mean. *See arithmetic mean.*

Median. A measure of central tendency. The value of the middle item in an uneven number of items arranged or arrayed according to size; the arithmetic average of the two central items in an even number of items similarly arranged.

Median absolute deviation. The median of the absolute deviations from the median. In a symmetrical distribution, the measure approximates one-half the IQR.

Median percent deviation. The median of the absolute percent deviations from the median; calculated by dividing the median absolute deviation by one-hundredth of the median.

Nonparametric statistics. *See* **distribution-free statistics.**

Nonsampling error. The error reflected in ratio study statistics from all sources other than sampling error. While nonsampling error is unavoidable due to the inefficiencies inherent in real property markets, the imperfections of the appraisal process, and the imperfections of conducting ratio studies, all practicable steps must be taken to minimize nonsampling error in ratio studies.

Normal distribution. A theoretical distribution often approximated in real world situations. It is symmetrical and bell-shaped; 68 percent of the observations occur within one standard deviation of the mean and 95 percent within two standard deviations of the mean.

Observation. One recording or occurrence of the value of a variable, for example, one sale ratio among a sample of sales ratios.

Outliers. Observations that have unusual values, that is, differ markedly from a measure of central tendency. Some outliers occur naturally; others are due to data errors.

Parameter. Numerical descriptive measure of the population, for example, the arithmetic mean or standard deviation. Parameters are generally unknown and estimated from statistics calculated from a sample of the population.

Parametric statistics. Statistics whose interpretation or reliability depends on the distribution of the underlying data. *See also* **distribution-free statistics.**

Percentile. The values that divide a set of data into specified percentages when the data are arrayed in ascending order. The tenth percentile includes the lowest 10 percent of the values, the twentieth percentile includes the lowest 20 percent of the values, and so forth.

Personal property. *See* **property.**

Plottage value. The excess of the value of a large parcel of land formed by assemblage over the sum of the values of the unassembled parcels.

Point estimate. A single numerical value that can be used to estimate a population parameter. It is calculated on the basis of information collected from a sample. Point estimates are generally constructed to provide the best unbiased estimate of the population parameter consistent with the sample data. However, the point estimate is only an estimate, and is unlikely to have the same value as the population parameter. (See **Confidence interval** and **Reliability** for discussion of precision of the sampling process.)

Points. Prepaid interest on a loan; one point is equal to 1 percent of the amount of the loan. It is common to deduct points in advance of the loan, so that an individual pays interest on 100 percent of the loan but gets cash on, say, only 99 percent.

Population. All the items of interest, for example, all the properties in a jurisdiction or neighborhood; all the observations in a data set from which a sample may be drawn.

Precision. The level of detail in which a quantity or value is expressed or represented. It can be characterized as the number of digits used to record a measurement. A high level of represented precision may be used to imply a greater level of accuracy; however, this relationship may not be true. Precision also relates to the quality of an operation or degree of refinement by which results are obtained. A method of measurement is considered precise if repeated measurements yield the same or nearly the same numeric value. *See also* **accuracy and statistical precision.**

PRB. *See* **coefficient of price-related bias.**

PRD. *See* **price-related differential.**

Price. The amount asked, offered, or paid for a property. (See USPAP [2004] for additional comments.)

Price-related differential. The mean divided by the weighted mean. The statistic has a slight bias upward. Price-related differentials above 1.03 tend to indicate assessment regressivity; price-related differentials below 0.98 tend to indicate assessment progressivity.

Progressivity. *See* **assessment progressivity (regressivity).**

Property. An aggregate of things or rights to things. These rights are protected by law. There are two basic types of property: real and personal. Real property consists of the interests, benefits, and rights inherent in the ownership of land plus anything permanently attached to the land or legally defined as immovable; the bundle of rights with which ownership of real estate is endowed. To the extent that “real estate” commonly includes land and any permanent improvements, the two terms can be understood to have the same meaning. Also called *realty*. Personal property is defined as those items that generally are movable or all items not specifically defined as real property. Many states include as personal property the costs associated with placing personal property in service, such as sales tax, freight, and installation. Installation items include, but are not limited to, wiring, foundations, hookups, and attachments. Two commonly used tests for distinguishing real and personal property are (1) the intent of the parties and (2) whether the item may be removed from the real estate without damage to either.

Qualified sale. A property transfer that satisfies the conditions of a valid sale and meets all other technical criteria for inclusion in a ratio study sample. If a property has undergone significant changes in physical characteristics, use, or condition in the period between the assessment date and sale date, it would not technically qualify for use in ratio study.

Quartiles. The values that divide a set of data into four equal parts when the data are arrayed in ascending order. The first quartile includes the lowest quarter of the data, the second quartile, the second lowest quarter, and so forth.

Random sample. A sample of n items selected from a population in such a way that each sample of the same size is equally likely. This also includes the case in which each element in the sample has an equal chance of being selected.

Range. (1) The maximum value of a sample minus the minimum value. (2) The difference between the maximum and minimum values that a variable may assume.

Ratio study. A study of the relationship between appraised or assessed values and market values. Indicators of market values may be either sales (sales ratio study) or independent “expert” appraisals (appraisal ratio study). Of common interest in ratio studies are the level and uniformity of the appraisals or assessments. *See also level of appraisal and level of assessment.*

Real property. *See property.*

Regressivity. *See assessment regressivity (regressivity).*

Regressivity index. *See price-related differential.*

Reliability. In a sampling process, the extent to which the process yields consistent population estimates. Ratio studies typically are based on samples. Statistics derived from these samples may be more or less likely to reflect the true condition in the population depending on the reliability of the sample. Representativeness, sample size, and sample uniformity all contribute to reliability. Formally, reliability is measured by sampling error or the width of the confidence interval at a specific confidence level relative to the central tendency measure.

Representative sample. A sample of observations from a larger population of observations, such that statistics calculated from the sample can be expected to represent the characteristics of the population being studied.

Sale price. (1) The actual amount of money exchanged for a unit of goods or services, whether or not established in a free and open market. An indicator of market value. (2) Loosely used synonymously with “offering” or “asking price.”

Sale ratio. The ratio of an appraisal (or assessed) value to the sale price or adjusted sale price of a property.

Sales chasing. Sales chasing is the practice of using the sale of a property to trigger a reappraisal of that property at or near the selling price. If sales with such appraisal adjustments are used in a ratio study, the practice causes invalid uniformity results and causes invalid appraisal level results, unless similar unsold parcels are reappraised by a method that produces an appraisal level for unsold properties equal

to the appraisal level of sold properties. (2) By extension, any practice that causes the analyzed sample to misrepresent the assessment performance for the entire population as a result of acts by the assessor’s office. A subtle, possibly inadvertent, variety of sales chasing occurs when the recorded property characteristics of sold properties are differentially changed relative to unsold properties. Then the application of a uniform valuation model to all properties results in the recently sold properties being more accurately appraised than the unsold ones.

Sales ratio study. A ratio study that uses sales prices as proxies for market values.

Sample. A set of observations selected from a population. If the sample was randomly selected, basic concepts of probability may be applied.

Sampling error. The error reflected in ratio study statistics that results solely from the fact that a sample of the population is used rather than a census of the population.

Scatter diagram or scatter plot. A graphic means of depicting the relationship or correlation between two variables by plotting one variable on the horizontal axis and one variable on the vertical axis. Often in ratio studies it is informative to determine how ratios are related to other variables. A variable of interest is plotted on the horizontal axis and ratios are plotted on the vertical axis.

Significance. A measure of the probability that an event is attributable to a relationship rather than merely the result of chance.

Skewed. The quality of a frequency distribution that makes it asymmetrical. Distributions with longer tails on the right than on the left are said to be skewed to the right or to be positively skewed. Distributions with longer tails to the left are said to be skewed to the left or to be negatively skewed.

Standard deviation. The statistic calculated from a set of numbers by subtracting the mean from each value and squaring the remainders, adding together all the squares, dividing by the size of the sample less one, and taking the square root of the result. When the data are normally distributed, the percentage of observations can be calculated within any number of standard deviations of the mean from normal probability tables. When the data are not normally distributed, the standard deviation is less meaningful and the analyst should proceed cautiously.

Standard error. A measure of the precision of a measure of central tendency; the smaller the standard error, the more reliable the measure of central tendency. Standard errors are used in calculating a confidence interval about the arithmetic mean and the weighted mean. The standard error of the sample mean is the standard deviation divided by the square root of the sample size.

Statistical accuracy. The closeness between the statistical estimate and the true (but unknown) population parameter value it was designed to measure. It is usually characterized in terms of error or the potential significance of error and can be decomposed into sampling error and nonsampling error components. Accuracy can be specified by the level of confidence selected for a statistical test. *See also accuracy.*

Statistical precision. A reference to how closely the survey results from a sample can reproduce the results that would be obtained from the entire population (a complete census). The amount by which a sample statistic can vary from the true population parameter is due to error. Even if all the sample data are perfectly accurate, random (sampling) error affects statistical precision (measured by the standard error or standard deviation). The dispersion of ratios in the population and the sample size have a controlling influence over the precision of any statistical estimate. When the reliability of a statistical measure is being evaluated, narrower confidence intervals have greater precision. *See also precision.*

Statistics. Numerical descriptive data calculated from a sample, for example, the median, mean, or COD. Statistics are used to estimate corresponding measures, termed parameters, for the population.

Stratify. To divide, for purposes of analysis, a sample of observations into two or more subsets according to some criterion or set of criteria.

Stratum, strata (pl.). A class or subset that results from stratification.

Time-adjusted sale price. The price at which a property sold adjusted for the effects of price changes reflected in the market between the date of sale and the date of analysis.

Trimmed mean. The arithmetic mean of a data set identified by the proportion of the sample that is trimmed from each end of the ordered array. For example, a 10 percent trimmed mean of a sample of size ten is the average of the eight observations remaining after the largest and smallest observations have been removed.

Value. (1) The relationship between an object desired and a potential owner; the characteristics of scarcity, utility, desirability, and transferability must be present for value to exist. (2) Value may also be described as the present worth of future benefits arising from the ownership of real or personal property. (3) The estimate sought in a valuation. (4) Any number between positive infinity and negative infinity.

Variable. An item of observation that can assume various values, for example, square feet, sales prices, or sales ratios. Variables are commonly described by using measures of central tendency and dispersion.

Weighted mean; weighted average. An average in which each value is adjusted by a factor reflecting its relative importance in the whole before the values are summed and divided by their number.

Weighted mean ratio. Sum of the appraised values divided by the sum of the sales prices (or independent estimates of market value), which weights each ratio in proportion to the sale price (or independent estimate of market value).