Birch, J.W. and M.A. Saunderman. 1997. Testing for outliers in sales and appraisal data. *Assessment Journal* 4(4):31–42.

Bonett, D.G. and E. Seier. 2006. Confidence intervals for a coefficient of dispersion in nonnormal distributions. *Biometrical Journal*. 48(1):144–148

Conover, W. J. 1980. *Practical nonparametric statistics*. New York: John Wiley & Sons.

D'Agostino, R.B., A. Belanger, and R.B. D'Agostino, Jr. 1990. A suggestion for using powerful and informative tests for normality. *The American Statistician* 44(4):316–21.

Denne, R.C. 2011. The PRB and other potential successors to the flawed PRD as a measure of vertical assessment inequity. *Fair & Equitable* 9(11): 3–10.

Dornfest, A.S., A. Chizewsky, and P. Davis. 2004. Alternate methods of addressing ratio study outliers. *Journal of Property Tax Assessment and Administration* 1(4):5–14.

Dornfest, A.S. 1993. Mass appraisal performance evaluation: Strategies for painless implementation. *Assessment Digest* 15(1):2–11.

Dornfest, A.S. 2000. State and provincial ratio study practices: 1997 Survey results. *Assessment Journal* 4(6):23–67.

Dornfest, A.S. 1990. Perspectives on ratio studies: The rural state. *Assessment Digest* 12(3):17–21.

Gloudemans, R.J. 1999. *Mass appraisal of real property.* Chicago: International Association of Assessing Officers.

Gloudemans, R.J. 1990. Adjusting for time in computer-assisted mass appraisal. *Property Tax Journal* 9(1):83–99.

Gloudemans, R.J. 2000. An empirical evaluation of central tendency measures. *Assessment Journal* 7(1):21–27.

Gloudemans, R.J. 2001. Confidence intervals for the COD: Limitations and solutions. *Assessment Journal* 8(6):23–27.

Gloudemans, R.J. 2011. The coefficient of price-related bias: A measure of vertical equity. *Fair & Equitable* 9(8): 3–8.

Groves, R.M. 2004. *Survey errors and survey costs*. New York: John Wiley & Sons.

International Association of Assessing Officers (IAAO). 2003. *Standard on facilities, computers, equipment and supplies*. Chicago: International Association of Assessing Officers.

International Association of Assessing Officers (IAAO). 1977. *Analyzing assessment equity*. Chicago: International Association of Assessing Officers.

Jacobs, T. 1986. Assessment quality control. *Assessment Digest* 8(4):8–13.

Benmamoun, M. 2006. Bootstrap confidence intervals and Gloudemans' COD tolerance test using SPSS and Stata. *Journal of Property Tax Assessment and Administration* 3(4):56–61.

Mendenhall, W., J.E. Reinmuth, and R.J. Beaver. 1993. *Statistics for management and economics*, 7th ed. Belmont, CA: Duxbury Press.

Neave, H. R., and P. L. Worthington. 1988. *Distribution-free tests*. London, England, and Boston, MA: Unwin Hyman.

Neter, J., W. Wasserman, and G.A. Whitmore. 1987. *Applied statistics*. 3rd ed. Boston, MA: Allyn and Bacon, Inc.

Snedecor, G.W., and W.G. Cochran. 1967. *Statistical methods,* 6th ed. Ames Iowa: Iowa State University Press.

Whorton, E.B. Jr. 2003. Should sales ratio studies be used for equalizing state funds for schools? *Assessment Journal.* 10(2):29–40.

Wu, K. and R. Baker, Jr. 2000. Using Tukey line in support of PRD. *Assessment Journal* 7(5):41–50.

Wu, K., and R. Baker, Jr. 2000. Evaluating assessment performance using a composite index: A tool for communication. *Assessment Journal* 7(1):29–33.

Additional readings on ratio studies can be found at LibraryLink, IAAO's online catalog of resources, and http://www.iaao.org. Many Web sites offer good information on statistics. Because Web site addresses change frequently, they are not listed here.

Appendix A. Sales Validation Guidelines

A.1 Sources of Sales Data

The best sources of sales data are copies of deeds or real estate transfer affidavits containing the full consideration and other particulars of the sale. Assessing officers in jurisdictions without laws mandating full disclosure of sales data to assessing officials work under a severe handicap and should seek legislation that provides for such disclosure.

1. *Real estate transfer document*s. These documents are (1) copies of deeds and land contracts, (2) copies of real estate transfer affidavits, and (3) closing statements.

2. *Buyers and seller*s. Buyers and sellers of real property can be contacted directly to secure or confirm sales data. Means of contact include sales questionnaires, telephone interviews, and personal interviews.

3. *Third-party source*s. Third-party sources include multiple listing agencies, real estate brokers and agencies, government and private fee appraisers, attorneys, appraisal organizations, and others. Of particular value are those individuals or agencies that publish lists of sales or provide sales in an electronic format.

A.2 Information Required

The following data are needed to make any necessary adjustments to sales prices, compute sales ratios, and update ownership information.

1. *Full consideration involved.* This is the total amount paid for the property, including the cash down payment and amounts financed. The sale price is the most essential item of information concerning the sale, and its accuracy must be carefully scrutinized. In many jurisdictions it is common practice in deeds of conveyance to state considerations in such terms as "one dollar plus other due and just consideration." These amounts are rarely the actual selling price and should be ignored in favor of information from the buyer and seller or other reliable source.

2. *Names of buyer and selle*r. This information permits the assessor to maintain a current record of the owners of all property in the jurisdiction. Transfer documents often refer to the buyer as the grantee or transferee and to the seller as the grantor or transferor.

3. *Addresses, phone numbers, and other contact information of buyer and seller or their legal designe*e. This information helps to identify more positively the parties to the sale. If the buyer will not reside at the property, the buyer's address may be needed for future correspondence. If the seller has established a new address, this information will aid the assessor in contacting the seller regarding the sale.

4. *Relationship of buyer and selle*r. It is important to know whether the buyer and seller are related individuals or corporate affiliates because such sales often do not reflect market value.

5. *Legal description, address, and parcel identifie*r. If each parcel is assigned a unique parcel identifier and if this number is noted on the document at the time it is recorded, then the assessor can locate the parcel in the files directly. If not, the legal description or street address is essential to locate the parcel.

6. *Type of transfe*r. It is crucial to identify whether or not a sale is an "arm's-length" transfer. Therefore, if the sources of sales data do not include copies of deeds, the type of deed should be specifically required.

7. *Time on the marke*t. Sales that have been exposed to the open market too long, not long enough, or not at all may not represent market value.

8. *Interest transferre*d. The appraiser must identify whether or not the entire bundle of rights (fee simple) to the property has transferred. For example, in some transactions, only a life tenancy ("life estate") may be conveyed, or the seller may retain mineral or other rights to the property. Similarly, the sale price of a property encumbered by a long-term lease may not reflect the market value of the fee simple estate in the property.

9. *Type of financin*g. In analyzing the sale, it is helpful to know the amount of down payment; the type, remaining amount, and interest rates of notes secured by mortgages or deeds of trust assumed by the buyer; and the value of any stocks, bonds, notes, or other property passed to the seller. It is also important to know whether the sale conveys title to the property or that it is a land contract, in which title is not conveyed until some time in the future, typically several years.

10. *Personal propert*y. A sales ratio study requires knowledge of the amount paid for the real

property. The sale document ideally would note the type and value of any significant personal property items included in the transaction.

11. *Date of transfer*. This is the date on which the sale was closed or completed. The date the deed or other transfer document was recorded can be used as a surrogate, provided there was no undue delay in the recording. If there has been a delay in recording, the date of the deed or transfer instrument should be used.

12. *Instrument number*. This number, as well as the record or deed book and page, indicates where the deed is located in the official records and thus can be important in researching sales or leases and identifying duplication.

The data noted above should be maintained in a separate data file or the sale history file component of a CAMA system. In addition, the file should include additional information useful for stratification and other analytical purposes. Sales data files should reflect the physical characteristics of the property when sold. If significant legal, physical, or economic changes have occurred between the sale date and the assessment date, the sale should not be used for ratio studies. (The sale may still be valid for mass appraisal modeling by matching the sale price against the characteristics that existed on the date of sale.)

A.3 Confirmation of Sales
A.3.1 Importance of Confirmation
The usefulness of sales data is directly related to the completeness and accuracy of the data. Sales data should be routinely confirmed or verified by contacting buyers, sellers, or other knowledgeable participants in the transaction. In general, the fewer the sales in a stratum, the less common or more complex the type of property, and the more atypical the sale price, the greater the effort should be to confirm the particulars of the sale. With larger sample sizes, it may be sufficient to confirm single-family residential sales by audit or exception.

A.3.2 Methods of Confirmation
In general, the completeness and accuracy of sales data are best confirmed by requesting the particulars of a sale from parties to the sale. If a transfer document is not required, questionnaires after the sale can be used. A sales questionnaire, which requests the type of information listed in Section A.2, is one practical means of confirming sales. Telephone or personal interviews can be more comprehensive than mailed questionnaires. Forms with space to record the same types of information should be used for such interviews. Appendix H contains a model sale confirmation questionnaire (additional sample sales questionnaires and interview forms can be found in *Improving Real Property Assessment* [IAAO 1978, 95–104]).

Mailed sales questionnaires should be as concise as possible and should include

- a postage-paid return envelope
- official stationery
- purpose of the questionnaire
- contact person
- authorized signature

Forms designed for telephone interviews should include the name and phone number of the contact person. Such forms also should include the date and name of the person conducting the interview along with the number of attempts made to contact a party to the sale.

A.4 Screening Sales
Sales used in a ratio study must be screened to determine whether they reflect the market value of the real property transferred. Specific objectives of sales screening are as follows:

- to ensure that sales prices reflect to the maximum extent possible only the market value of the real property transferred and not the value of personal property, financing, or leases
- to ensure that sales that occurred only during the period of analysis are used
- to ensure that sales are excluded from the ratio study only with good cause (e.g., when they compromise the validity of the study)

Every arm's-length, open-market sale that appears to meet the conditions of a market value transaction should be included in the ratio study unless one of the following occurs:

- Data for the sale are incomplete, unverifiable, or suspect.
- The sale fails to pass one or more specific tests of acceptability.
- A sufficiently representative sample of sales that occurred during the study period can be randomly selected to provide sufficiently reliable statistical measures.

The sales reviewer should take the position that all sales are candidates as valid sales for the ratio study unless sufficient and compelling information can be documented to show otherwise. If sales are excluded without substantiation, the study may appear to be subjective. Reason codes can be established for invalid sales.

No single set of sales screening rules or recommendations can be universally applicable for all uses of sales data or under all conditions. Sales screening guidelines and procedures should be consistent with the provisions of the

value definition applicable to the jurisdiction. Appraisers must use their judgment, but should not be arbitrary. To help analysts make wise and uniform judgments, screening procedures should be in writing. Each sales analyst should be thoroughly familiar with these procedures as well as with underlying real estate principles (Tomberlin 2001).

A.4.1 Sales Generally Invalid for Ratio Studies

The following types of sales are often found to be invalid for ratio studies and can be automatically excluded unless a larger sample size is needed and further research is conducted to determine that sales are open-market transactions.

1. *Sales involving government agencies and public utilitie*s. Such sales can involve an element of compulsion and often occur at prices higher than would otherwise be expected.

2. *Sales involving charitable, religious, or educational institution*s. A sale to such an organization can involve an element of philanthropy, and a sale by such an organization can involve a nominal consideration or restrictive covenants.

3. *Sales involving financial institution*s. A sale in which the lienholder is the buyer can be in lieu of a foreclosure or a judgment and the sale price can equal the loan balance only.

4. *Sales between relatives or corporate affiliate*s. Sales between relatives are usually non-open-market transactions and tend to occur at prices lower than would otherwise be expected.

5. *Sales settling an estat*e. A conveyance by an executor or trustee under powers granted in a will may not represent fair market value, particularly if the sale takes place soon after the will has been filed and admitted to probate in order to satisfy the decedent's debts or the wishes of an heir.

6. *Forced sale*s. Such sales include those resulting from a judicial order. The seller in such cases is usually a sheriff, receiver, or other court officer.

7. *Sales of doubtful titl*e. Sales in which title is in doubt tend to be below market value. When a sale is made on other than a warranty deed, there is a question of whether the title is merchantable. Quit claim deeds and trustees' deeds are examples.

A.4.2 Sales with Special Conditions

Sales with special conditions can be open-market sales but must be verified thoroughly and used with caution in ratio studies.

1. *Trades.* In a trade, the buyer gives the seller one or more items of real or personal property as all or part of the full consideration. If the sale is a pure trade with the seller receiving no money or securities, the sale should be excluded from analysis. If the sale involves both money and traded property, it may be possible to include the sale in the analysis if the value of the traded property is stipulated, can be estimated with accuracy, or is small in comparison to the total consideration. However, transactions involving trades should be excluded from the analysis whenever possible, particularly when the value of the traded property appears substantial.

2. *Partial interest*s. A sale involving the conveyance of less than the full interest in a property should be excluded from the analysis unless several sales of partial interests in a single property take place at the same time and the sum of the partial interests equals the fee-simple interest. Then the sum of the sales prices of the partial interests can sometimes be assumed to indicate the sale price of the total property. At other times, however, the purchase of such partial interests is analogous to plottage value in which a premium may have been paid.

3. *Land contract*s. Land contracts and other installment purchase arrangements in which title is not transferred until the contract is fulfilled require careful analysis. Deeds in fulfillment of a land contract often reflect market conditions several years in the past, and such dated information should be excluded from analysis. Sales data from land contracts also can reflect the value of the financing arrangements. In such instances, if the transaction is recent, the sale price should be adjusted for financing (see section A.5.2).

4. *Incomplete or unbuilt common propert*y. Sales of condominium units and of units in planned unit developments or vacation resorts often include an interest in common elements (for example, golf courses, clubhouses, or swimming pools) that may not exist or be usable on the date of sale or on the assessment date. Sales of such properties should be examined to determine whether prices might be influenced by promises to add or complete common elements at some later date. Sales whose prices are influenced by such promises should be excluded from the analysis, or the sales prices should be adjusted to reflect only the value of the improvements or amenities in existence on the assessment date.

5. *Auction*s. In general, auction sales of real property tend to be at the lower end of the price spectrum. Auction sales that have been well-advertised and well-attended may be valid for consideration in ratio studies. The seller also must have the option to set a minimum bid on the property or the right of refusal on all bids *(with reserv*e) in order for the sale to be considered valid.

A.4.3 Multiple-Parcel Sales

A multiple-parcel sale is a transaction involving more than one parcel of real property. These transactions present special considerations and should be researched and analyzed before being used in ratio studies.

If the appraiser needs to include multiple-parcel sales, he or she should first determine whether the parcels are contiguous and whether the sale comprises a single economic unit or multiple economic units. Regardless of whether the parcels are contiguous, any multiple-parcel sale that also involves multiple economic units generally should not be used in ratio studies because of the likelihood that these sales include some plottage value or some discount for economies of scale, unless adequate adjustments for these factors can be made to the sale price.

A.4.4 Acquisitions or Divestments by Large Property Owners

Acquisitions or divestments by large corporations, pension funds, or real estate investment trusts (REITs) that involve multiple parcels typically should be rejected for ratio study purposes.

A.4.5 IRS 1031 Exchanges

Internal Revenue Service (IRS) Regulation 1031 stipulates that investment properties can be sold on a tax-deferred basis if certain requirements are met. Sale transactions that represent Section 1031 exchanges should be analyzed like any other commercial transaction and, absent conditions that would make the sale price unrepresentative of market value, should be regarded as valid.

A.4.6 Internet Marketing

Property that sells on the Internet and meets the criteria of being an open-market, arm's-length transaction should be included as a valid transaction in a ratio study. Brokerage and realty firms are using the Internet as an additional method to advertise and market their inventory of property.

A.4.7 Inaccurate Sale Data

Sale information should never be considered absolutely trustworthy. Jurisdictions can reduce the problem by requiring a sale verification questionnaire (see Appendix H). There should be statutory penalties for persons who falsify information.

A.5 Adjustments to Sale Prices

Sale prices used in ratio studies may need to be adjusted for financing, assumed long-term leases, personal property, gift programs, and date of sale. This is especially true for nonresidential properties. The real property tax is based on the market value of real property alone as of a specific date. This value may not be the same as investment value (that is, the monetary value of a property to a particular investor) and does not include the value of personal property or financing arrangements.

If adjustments for more than one purpose are to be made, they should be made in the following order:

1. adjustments that convert the price to a better representation of the market value as of the date of sale (These include adjustments for financing and assumed long-term leases.)

2. adjustments that develop or isolate the price paid for taxable real property (These include adjustments for personal property received by the buyer, property taken in trade by the seller, the combination of partial interest sales, and incomplete or unbuilt common property.)

3. adjustments for differences in market value levels between the date of sale and the date of analysis

Procedures for adjusting sales prices should be documented and the adjustment factors supported by market data. These requirements imply an ongoing study of local real estate prices, interest rates, and financing practices. Unsubstantiated or blanket adjustments can jeopardize the acceptance accorded a ratio study by making it appear subjective.

A.5.1 Adjustments for Financing

When financing reflects prevailing market practices and interest rates, sales prices require no adjustment for financing. Adjustments should be considered in the following instances:

1. The seller and lender are the same party and financing is not at prevailing market rates.

2. The buyer assumes an existing mortgage at a non-market interest rate. As with personal property, the preferred means of adjusting for financing is by individual parcel. In this instance and no. 1 above, downward adjustments are warranted when (1) the loan appears to be well secured and the contract interest rate is less than the market interest rate, or (2) the loan appears not to be well secured and the contract interest

rate is lower than that required by the market for a loan of equal risk. The amount of adjustment can be computed by capitalizing the difference between monthly payments based on the required market interest rate and those based on the actual interest rate. Market analysis using paired sales (sales of similar properties, some with and some without conventional financing) or statistical techniques can correct for such factors.

3. The seller pays "points" (a percentage of the loan amount). *(Points paid by the borrower are part of the down payment and do not require adjustment.)* When the seller pays points, the sale price should be adjusted downward by the value of the points.

4. The property is sold under a gift program. Gift programs are a type of creative financing for qualified buyers by certain lending institutions that provide the buyer with additional monies to use as part of a down payment or for property improvements. This program is typically associated with low-value properties and can be difficult to discover without a validation questionnaire and/or telephone interview. The gift amount is added to the actual sale price of the property; however, the seller is never in receipt of the gift amount. This gift amount must be deducted from the actual sale price of the real estate prior to statistical analysis.

Adjustments for financing require data on actual and market interest rates, the amount of the loan, and the term and amortization provisions of the loan. Obtaining and properly analyzing such data, as well as estimating the extent to which the market actually capitalizes non-market financing, are difficult and time-consuming and require specialized skills.

A.5.2 Adjustments for Assumed Leases

The sale price of a property encumbered by a long-term lease of at least three years should be adjusted as follows:

- If the contract rent differs significantly from market rent, then the sales price should be adjusted by the difference between the present worth of the two income streams.

- If the contract rent exceeds market rent, the present worth of the difference in the two income streams should be subtracted from the sale price.

- If the contract rent is less than current market rent, the present worth of the difference in the two income streams should be added to the sale price.

A.5.3 Adjustments for Personal Property

Sales screening includes determining the contributory value of any significant personal property included in the sale. Personal property includes such tangibles as machinery, furniture, and inventories and such intangibles as franchises, licenses, and non-compete agreements. Ordinarily, it is not necessary to consider goodwill, going-concern value, business enterprise value, or the like, unless the value of these intangible assets has been itemized in a sales contract or a formal appraisal has been prepared by either party.

It is necessary to decide whether each item included in the sale should be classified as real or personal property. (See *Standard on Valuation of Personal Property* [IAAO 2005], which provides guidance on classification of property as real or personal.)

Sale prices should be adjusted by subtracting the contributory value of personal property received by the buyer. Ordinarily window treatments, outdated models of free-standing appliances, and common-grade used furniture included with residential property do not usually influence the sale price of real property and do not require an adjustment unless the items were specifically broken out in the contract as personal property included in the sale price.

If the value of personal property appears to be substantial (10 percent for residential, 25 percent for commercial), the sale should be excluded as a valid sale in statistical analysis unless the sample size is small.

A.5.4 Adjustments for Time

There should be a program to track changes in price levels over time and adjust sale prices for time as required. This step is an important component of a ratio study. Time adjustments must be based on market analysis and supported with appropriate documentation.

Valid time-adjustment techniques are as follows:

- tracking sales and appraisal ratios over time

- including date-of-sale as a variable in regression or feedback models

- analyzing re-sales

- comparing per-unit values over time in homogeneous strata, such as a subdivision or condominium complex

- isolating the effect of time through paired sales analysis

- statistically supported time trend analysis studies

These techniques are discussed in Gloudemans (1990; 1999), *Property Appraisal and Assessment Administration* (IAAO 1990, Appendix 5-3), and *Improving Real Property Assessment* (IAAO 1978, section 4.6). If sales

prices have generally been rising, ratios for sales that occurred after the assessment date tend to understate the overall level of appraisal. Similarly, sales ratios for sales that occurred before the assessment date tend to overstate the level of appraisal. If prices are generally declining, an opposite pattern results. When tracking ratios over time (using the inverse ratio technique) for determining time adjustments, it is important that ratios for chased sales be excluded, since there is no correlation of such sales ratios with the date of sale.

Changes in price levels should be monitored and time adjustments made by geographic area and type of property, because different segments of the market tend to change in value at different rates.

Oversight agencies can make any appropriate time adjustments after making all other adjustments.

A.5.5 Other Adjustments
Adjustments to sales prices should not be made for real estate sales and brokerage commissions; closing costs, such as attorney's fees, transfer taxes, and title insurance; and current or delinquent property taxes. Exceptions to this general rule occur when the buyer agrees to pay real estate commissions and delinquent property taxes, in which case the amounts of the payments should be added to the sale price if not already included in the sale amount. Other exceptions occur when the seller agrees to pay expenses normally paid by the buyer. Such expenses include loan origination fees and repair allowances. Loan origination fees paid by the seller should be deducted from the sale price. Repair allowances should be deducted from the sales price only if the property was in an unrepaired state on the appraisal date, but sold at a higher price reflecting the value of the repairs. If the sale occurred before the appraisal date and the repairs were made prior to that date, no adjustment should be made (Knight, Miceli, and Sirmans 2000).

A.5.6 Special Assessments
Special assessments are used to finance capital improvements or provide services adjacent to the properties they directly benefit. Typically, the property owner is obligated to make annual payments of principal and interest to a local unit of government over a specified number of years. The sale price of a property encumbered by a special assessment can require adjustment if the current balance of the defrayed amount is significant. The sale price can be adjusted upward to account for this lien. If the effect on market value is significant and can be ascertained, an adjustment should be made.

Appendix B. Outlier Trimming Guidelines

B.1 Identification of Ratio Outliers

It is first necessary to determine a procedure to identify outliers. Outlier identification based on the interquartile range (IQR) uses order statistics (see table B-1) and has been shown to be robust for a wide variety of distributions (Iglewicz and Hoaglin 1993; Barnett and Lewis 1994). The term outlier is often associated with ratios that fall outside 1.5 multiplied by the IQR. A factor of 3.0 X IQR often is chosen to identify extreme outliers. Other outlier identification procedures are found in statistical literature and can be used. Outlier identification and trimming should follow the sales validation process and precede the calculation of ratio statistics and related tests or analyses.

The example in table B-1 demonstrates the use of the 1.5 X IQR procedure to identify outlier ratios. The distribution of ratios often is skewed to the right; therefore, it may be preferable to apply appropriate transformations to the ratios prior to applying the IQR method. For example, the use of logarithmic transformations tends to identify fewer high and more low ratios as outliers.

B.2 Scrutiny of Identified Outliers

The preferred method of handling an outlier ratio is to subject it to additional scrutiny to determine whether the sale is a non-market transaction or contains an error in fact. If an error can be corrected (for example, data entry), the property should be left in the sample. If the error cannot be corrected or inclusion of the identified outlier would reduce sample representativeness, the sale should be excluded.

B.3 Outlier Trimming

Once outliers have been identified and scrutinized and any errors resolved, the next step is to exclude those that may unduly influence calculated statistical measures. For this reason, it is acceptable to trim outliers identified by recognized procedures (for cautionary notes on trimming small samples, see Tomberlin [2001] and Hoaglin, Mosteller, and Tukey [1983]. An example of such trimming is found in Table B-2. However, trimming of outliers using arbitrary limits, for example, eliminating all ratios less than 50 percent or greater than 150 percent, tends to distort results and should not be employed.

Detected outliers should be reported and can be treated in a variety of ways, including trimming (D'Agostino and Stephens 1986). If outliers are to be considered for removal, the analyst can select a procedure to trim all or just the extreme or influential outliers (see table B-2). If a trimming method has been used to reject ratios from the sample, this fact must be stated in the resulting statistical

Table B-1. A Distribution-Free Method for Locating Outliers
(The following procedure identifies outlier ratios that fall more than 1.5 times beyond the range of the middle 50 percent of the arrayed sample.)

Locating trim boundaries
Data set before trimming

Rank	Ratio (A/S)
1	0.611
2	0.756
3	0.762
4	0.853
5	0.867
6	0.909
7	0.925
8	0.944
9	1.014
10	1.052
11	1.178
12	1.367
13	1.850
14	2.500

Median ratio	**0.935**
COD	**32.271**

Steps to locate trim boundaries

1. *Locate the first quartile point*
   Formula to locate the first quartile:
   (0.25 x number of ratios) + 0.25
   (0.25 x 14 ratios) + 0.25 = 3.75
   3.75 is three-quarters between the third and fourth ranked ratios.
   Ratio 3 = 0.762
   Ratio 4 = 0.853
   Three-quarters between = (0.853 − 0.762) x 0.75 = 0.068
   The first quartile point = 0.762 + 0.068 = 0.830

2. *Locate the third quartile point*
   Formula to locate the third quartile
   (0.75 x number of ratios) + 0.75
   (0.75 x 14 ratios) + 0.75 = 11.25
   11.25 is one-quarter between the eleventh and twelfth ranked ratios.
   Ratio 11 = 1.178
   Ratio 12 = 1.367
   One-quarter between = (1.367 − 1.178) x 0.25 = 0.047
   The third quartile point = 1.178 + 0.047 = 1.225

3. *Compute the interquartile range*
   The distance between the first and third quartile = interquartile range
   1.225 − 0.830 = 0.395

4. *Establish the lower boundary*
   Lower trim point = first quartile − (interquartile range x 1.5 or 3.0)
   0.830 − (0.395 x 1.5) = 0.238,

5. *Establish the upper boundary*
   Upper trim point = (interquartile range x 1.5 or 3.0) + third quartile
   (0.395 x 1.5) + 1.225 = 1.818

Outliers identified:
1.850
2.500

Table B-2. Effects of Outlier Trimming
Outliers identified in Table B-1 trimmed

	After 1.5x trimming
Rank	**Ratio (A/S)**
1	0.611
2	0.756
3	0.762
4	0.853
5	0.867
6	0.909
7	0.925
8	0.944
9	1.014
10	1.052
11	1.178
12	1.367
Median ratio	0.917
COD	15.649

analysis. Outlier trimming is not mandatory; however, if outlier-trimming procedures are not used, sales with extreme or influential ratios must be thoroughly validated and determined to be highly trustworthy observations because they can play a pivotal role in the ratio study outcome.

B.4 Trimming Limitations

For some distributions, such as when the sample exhibits a high clustering around a specific ratio, the IQR outlier identification method is not appropriate. In such cases the IQR could be quite narrow, leading to the calculation of lower and upper boundaries for outliers and extremes that are quite close to the middle of the data. In such cases, ratios beyond those boundaries should not be automatically excluded, but instead reasonable judgment should be applied to exclude only true outliers or extremes. As one safeguard, analysts can refrain from automatically

deleting any "outliers" or "extremes" inside the boundaries where 95 percent (two standard deviations) of the observations would be expected to lie, assuming a normal distribution of data.

It is also appropriate to set maximum trimming limits. For small samples, no more than 10 percent (20 percent in the most extreme cases) of the ratios should be removed. For larger samples, this threshold can be lowered to 5 to 10 percent depending on the distribution of the ratios and the degree to which sales have been screened or validated. Trim limits should be developed in consideration of the extent of sales verification.

In general, IQR-based outlier identification should be undertaken in instances in which sample sizes are sufficient to preclude the aberrant results that can be expected when this procedure is applied to small, highly variable samples.

B.5 Analytical Use of Identified Outliers

After identification, scrutiny, and correction of errors associated with outliers, the procedure can be run again to identify any remaining apparent outliers. If outlier ratios tend to be concentrated in certain areas or other subsets of the sample, they can point directly to systematic errors in the appraisal process and should be stratified and reanalyzed if they are sufficiently representative.

B.6 Reporting Trimmed Outliers and Results

Ratio study reports or accompanying documentation should clearly state the basis for excluding outlier ratios. Statistics calculated from trimmed distributions, obviously, cannot be compared to those from untrimmed distributions or interpreted in the same way.

Return to Table of Contents

Appendix C.
Median Confidence Interval Tables for Small Samples

For small samples, tables C-1 and C-2 demonstrate use of a formula based upon the binomial distribution (Clapp 1989) to develop the lower and upper median confidence interval estimates. R_i is the ratio in an array ranked from the lowest ($i = 1$) to the highest (sorted in ascending order). Each confidence interval boundary typically falls between two ratios in the array. The interpolation factor is multiplied by the ratio value and the two are added together to obtain a weighted average. This method should be used for small samples with up to 30 observations (see tables C-1 and C-2). For larger samples the method found in *Property Appraisal and Assessment Administration* (IAAO 1990, p 609) may be used.

Example
Using data from table 1-4 (n =17 ratios) and a 95 percent confidence interval in table C-2:

Lower bound:

[0.695 (Ratio$_5$) X 0.9899] + [0.717 (Ratio$_6$) X 0.0101] = **0.695**

Upper bound:

[0.933 (Ratio$_{13}$) X 0.9899] + [0.895 (Ratio$_{12}$) X 0.0101] = **0.933**

Therefore, the 95% median ratio confidence interval in table 1-4 is from .695 to .933.

From Table 1-4. Demonstration Ratio Study Report

Rank	Parcel #	Appraised value	Sale price*	Ratio
1	9	$87,200	138,720	0.629
2	10	38,240	59,700	0.641
3	11	96,320	146,400	0.658
4	12	68,610	99,000	0.693
5	13	32,960	47,400	0.695
6	14	50,560	70,500	0.717
7	15	61,360	78,000	0.787
8	16	47,360	60,000	0.789
9	17	56,580	69,000	0.820
10	18	47,040	55,500	0.848
11	19	136,000	154,500	0.880
12	20	98,000	109,500	0.895
13	21	56,000	60,000	0.933
14	22	159,100	168,000	0.947
15	23	128,000	124,500	1.028
16	24	132,000	127,500	1.035
17	25	160,000	150,000	1.067

Date: 0/0/00.   *No outlier trimming*

* or adjusted sale price

Table C-1. 90% Confidence Interval Table

n	Lower Bound	Upper Bound
5	$.8800 \times R^1 + .1200 \times R^2$	$.8800 \times R^5 + .1200 \times R^4$
6	$.6333 \times R^1 + .3667 \times R^2$	$.6333 \times R^6 + .3667 \times R^5$
7	$.2286 \times R^1 + .7714 \times R^2$	$.2286 \times R^7 + .7714 \times R^6$
8	$.8643 \times R^2 + .1357 \times R^3$	$.8643 \times R^7 + .1357 \times R^6$
9	$.5667 \times R^2 + .4333 \times R^3$	$.5667 \times R^8 + .4333 \times R^7$
10	$.1067 \times R^2 + .8933 \times R^3$	$.1067 \times R^9 + .8933 \times R^8$
11	$.7855 \times R^3 + .2145 \times R^4$	$.7855 \times R^9 + .2145 \times R^8$
12	$.4282 \times R^3 + .5718 \times R^4$	$.4282 \times R^{10} + .5718 \times R^9$
13	$.9558 \times R^4 + .0442 \times R^5$	$.9558 \times R^{10} + .0442 \times R^9$
14	$.6511 \times R^4 + .3489 \times R^5$	$.6511 \times R^{11} + .3489 \times R^{10}$
15	$.2217 \times R^4 + .7783 \times R^5$	$.2217 \times R^{12} + .7783 \times R^{11}$
16	$.8261 \times R^5 + .1739 \times R^6$	$.8261 \times R^{12} + .1739 \times R^{11}$
17	$.4603 \times R^5 + .5397 \times R^6$	$.4603 \times R^{13} + .5397 \times R^{12}$
18	$.9735 \times R^6 + .0265 \times R^7$	$.9735 \times R^{13} + .0265 \times R^{12}$
19	$.6480 \times R^6 + .3520 \times R^7$	$.6480 \times R^{14} + .3520 \times R^{13}$
20	$.2072 \times R^6 + .7928 \times R^7$	$.2072 \times R^{15} + .7928 \times R^{14}$
21	$.8084 \times R^7 + .1952 \times R^8$	$.8084 \times R^{15} + .1952 \times R^{14}$
22	$.4156 \times R^7 + .5844 \times R^8$	$.4156 \times R^{16} + .5844 \times R^{15}$
23	$.9413 \times R^8 + .0587 \times R^9$	$.9413 \times R^{16} + .0587 \times R^{15}$
24	$.5884 \times R^8 + .4116 \times R^9$	$.5884 \times R^{17} + .4116 \times R^{16}$
25	$.1203 \times R^8 + .8797 \times R^9$	$.1203 \times R^{18} + .8797 \times R^{17}$
26	$.7371 \times R^9 + .2629 \times R^{10}$	$.7371 \times R^{18} + .2629 \times R^{17}$
27	$.3161 \times R^9 + .6839 \times R^{10}$	$.3161 \times R^{19} + .6839 \times R^{18}$
28	$.8687 \times R^{10} + .1313 \times R^{11}$	$.8687 \times R^{19} + .1313 \times R^{18}$
29	$.4831 \times R^{10} + .5169 \times R^{11}$	$.4831 \times R^{20} + .5169 \times R^{19}$
30	$.9876 \times R^{11} + .0124 \times R^{12}$	$.9876 \times R^{20} + .0124 \times R^{19}$

Table C-2. 95% Confidence Interval Table

n	Lower Bound	Upper Bound
6	$.9000 \times R^1 + .1000 \times R^2$	$.9000 \times R^6 + .1000 \times R^5$
7	$.6857 \times R^1 + .3143 \times R^2$	$.6857 \times R^7 + .3143 \times R^6$
8	$.3250 \times R1 + .6750 \times R^2$	$.3250 \times R^8 + .6750 \times R^7$
9	$.9222 \times R^2 + .0778 \times R^3$	$.9222 \times R^8 + .0778 \times R^7$
10	$.6756 \times R^2 + .3244 \times R^3$	$.6756 \times R^9 + .3244 \times R^8$
11	$.2873 \times R^2 + .7127 \times R^3$	$.2873 \times R^{10} + .7127 \times R^9$
12	$.8936 \times R^3 + .1064 \times R^4$	$.8936 \times R^{10} + .1064 \times R^9$
13	$.6056 \times R^3 + .3944 \times R^4$	$.6056 \times R^{11} + .3944 \times R^{10}$
14	$.1659 \times R^3 + .8341 \times R^4$	$.1659 \times R^{12} + .8341 \times R^{11}$
15	$.8218 \times R^4 + .1782 \times R^5$	$.8218 \times R^{12} + .1782 \times R^{11}$
16	$.4827 \times R^4 + .5173 \times R^5$	$.4827 \times R^{13} + .5173 \times R^{12}$
17	$.9899 \times R^5 + .0101 \times R^6$	$.9899 \times R^{13} + .0101 \times R^{12}$
18	$.7076 \times R^5 + .2924 \times R^6$	$.7076 \times R^{14} + .2924 \times R^{13}$
19	$.3059 \times R^5 + .6941 \times R^6$	$.3059 \times R^{15} + .6941 \times R^{14}$
20	$.8835 \times R^6 + .1165 \times R^7$	$.8835 \times R^{15} + .1165 \times R^{14}$
21	$.5479 \times R^6 + .4521 \times R^7$	$.5479 \times R^{16} + .4521 \times R^{15}$
22	$.0697 \times R^6 + .9303 \times R^7$	$.0697 \times R^{17} + .9303 \times R^{16}$
23	$.7381 \times R^7 + .2619 \times R^8$	$.7381 \times R^{17} + .2619 \times R^{16}$
24	$.3373 \times R^7 + .6627 \times R^8$	$.3373 \times R^{18} + .6627 \times R^{17}$
25	$.8958 \times R^8 + .1042 \times R^9$	$.8958 \times R^{18} + .1042 \times R^{17}$
26	$.5481 \times R^8 + .4519 \times R^9$	$.5481 \times R^{19} + .4519 \times R^{18}$
27	$.0677 \times R8 + .9323 \times R9$	$.0677 \times R20 + .9323 \times R19$
28	$.7221 \times R9 + .2779 \times R10$	$.7221 \times R20 + .2779 \times R19$
29	$.3063 \times R9 + .6937 \times R10$	$.3063 \times R21 + .6937 \times R20$
30	$.8709 \times R10 + .1291 \times R11$	$.8709 \times R21 + .1291 \times R20$

Appendix D. Coefficient of Price-Related Bias

The coefficient of price-related bias (PRB) is an index of vertical equity that quantifies the relationship between assessment-sales ratios (ASR) and value in percentage terms. A PRB of 0.043 indicates that, on average, assessment ratios increase by 4.3 percent whenever values increase by 100 percent (e.g., double or double again). The PRB has several technical advantages, including being less sensitive to outliers than the PRD, and also quantifies the statistical significance of observed relationships. Using table D-1 as an example, the measure is found as follows:

1. Compute a value proxy, "value," as 50 percent of sale price + 50 percent of assessed value. To ensure that assessed values and sales prices receive equal weight, assessed values can be divided by the median ratio before summing:

   Value = 0.50 × (AV/Median) + 0.50 × SP

   Where:
   AV= Assessed Value
   SP = Sale Price

Columns (5) and (6) illustrate the calculation. Computing a value proxy based on both assessed values and sales prices minimizes bias inherent in comparing ratios against either assessed values or sales prices alone (see, for example, Gloudemans and Almy 2010, pp 219, 229, 389–391).

2. Take the natural logarithm of the value proxy and divide by 0.693:

   Ln_Value = ln(value)/0.693

   This is shown in column (7) of table D-1. Taking logarithms converts the value proxy to a percentage basis, which substantially minimizes the impact of atypically high values (outliers) in the analysis. Dividing by 0.693 allows each increment of 1 to be interpreted as a change of 100 percent. (For example, ln(100,000)/0.693 = 16.613 and ln(200,000)/0.693 = 17.613).

3. Compute percentage differences from the median assessment ratio (column 8 of table D-1):

Table D-1. Illustration of PRB

(1)	(2)	(3)	(4)	(5)	(6) .5(3) + .5(5)	(7) Indep Var	(8) Dep Variable
Sale	AV	SP	ASR	AV/Med	"Value"	Ln(Value)/.693	(ASR − Med)/Med
1	116,700	114,500	1.019	128,267	121,383	16.893	0.120
2	130,300	121,000	1.077	143,215	132,107	17.015	0.184
3	130,200	133,900	0.972	143,105	138,503	17.083	0.069
4	145,500	139,000	1.047	159,921	149,461	17.193	0.151
5	134,100	145,000	0.925	147,392	146,196	17.161	0.016
6	153,900	156,500	0.983	169,154	162,827	17.317	0.081
7	143,400	161,100	0.890	157,613	159,357	17.286	−0.022
8	156,900	169,500	0.926	172,451	170,976	17.387	0.017
9	169,000	175,000	0.966	185,751	180,375	17.464	0.061
10	149,200	181,000	0.824	163,988	172,494	17.400	−0.094
11	160,100	188,900	0.848	175,969	182,434	17.481	−0.068
12	191,400	205,000	0.934	210,371	207,685	17.668	0.026
13	177,200	216,150	0.820	194,763	205,457	17.652	−0.099
14	205,500	219,000	0.938	225,868	222,434	17.767	0.031
15	206,500	235,000	0.879	226,968	230,984	17.821	−0.034
16	243,800	249,000	0.979	267,965	258,482	17.984	0.076
17	211,600	258,900	0.817	232,573	245,737	17.911	−0.102
18	242,500	263,000	0.922	266,536	264,768	18.018	0.013
19	258,400	305,900	0.845	284,012	294,956	18.174	−0.072
20	265,900	312,500	0.851	292,255	302,378	18.210	−0.065
21	305,700	336,000	0.910	336,000	336,000	18.362	0.000
22	291,600	360,000	0.810	320,502	340,251	18.380	−0.110
23	312,800	399,900	0.782	343,804	371,852	18.508	−0.140
24	352,200	418,500	0.842	387,109	402,805	18.624	−0.075
25	354,900	459,000	0.773	390,077	424,538	18.700	−0.150
Sum	5,209,300	5,923,250	22.578			PRB	−0.120
						Std Error	0.025
	Median	0.910		COD	0.075	t-value	−4.721
	Mean	0.903		PRD	1.027	d.f.	23
	WtdMean	0.879		Sales	25	Sig	0.000

Pct_Diff = (ASR – Median)/Median

Where:

PCT_Diff = Percentage Difference

ASR = Assessment-Sales Ratio

4. Regress (3) on (2):

Pct_Diff = $b0 + b1 \times$ Ln_Value

Because each increment of 1 in the independent variable represents a 100 percent change in value, the regression coefficient, $b1$, represents the corresponding percentage change in assessment ratios.

Figures D-1 and D-2 below contain plots of assessment ratios with assessed values and sales prices, respectively. Similarly, Figure D-3 is a plot of ratios against the value proxy and Figure D-4 plots percentage differences from the median ratio on logarithms of the value proxy divided by 0.693. In this case, all four plots show a regressive relationship. The PRB quantifies the relationship. As shown toward the bottom of table D-1, PRB = –0.120, meaning that ratios

decline by 12.0 percent when values double (and increase by 12.0 percent when values are halved). The relationship is significant at the 99.9 percent confidence level. The 95 percent confidence interval is −0.172 to −0.067.

To illustrate the relative insensitivity of the PRB to outliers, consider table D-2. Sales prices for the first 15 sales increase by increments of $50,000: from $50,000 for sale 1 to $750,000 for sale 15. The ratios alternate from 0.90, to 1.00, to 1.10. Since the first (lowest sale) has a ratio of 0.90 and the highest sale has a ratio of 1.10, there is minor progressivity. As shown in the upper half of table D-3, the COD is 6.7, the PRD is 0.992, and the PRB is 0.02, all good performance measures.

Now consider sale 16 in table D-2, which is a relative outlier with a sale price of $2,500,000 and ratio of 0.75. As shown in the lower half of table D-3, the PRD falls well outside of 0.98 to 1.03 and indicates regressivity. The PRB (as denoted in Table D-3 in the column entitled "Coefficients" and "B"), with a benign value of -0.011, is little affected by the outlier and is not statistically significant.
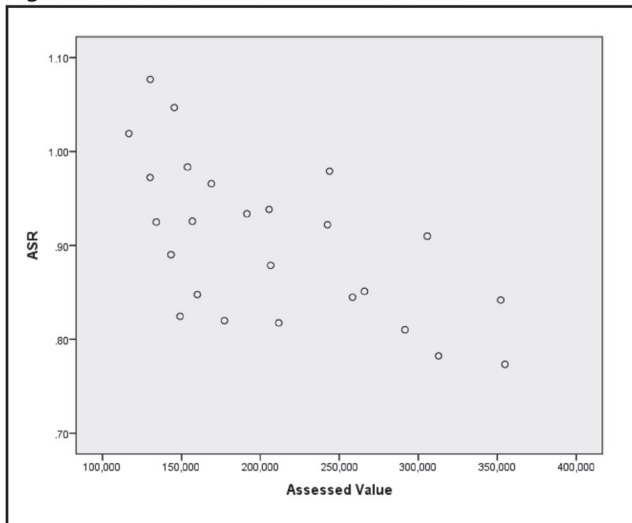
Figure D-1. Plot of Ratios with Assessed Value
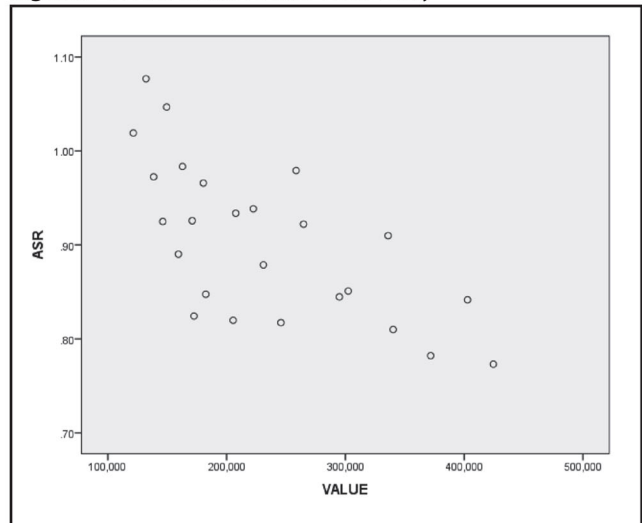

Figure D-3. Plot of Ratios with Value Proxy

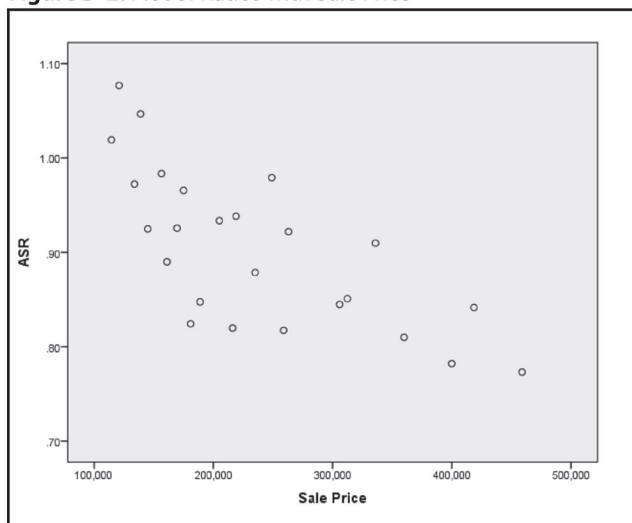**Figure D-2.** Plot of Ratios with Sale Price

Figure D-4. PRB Plot
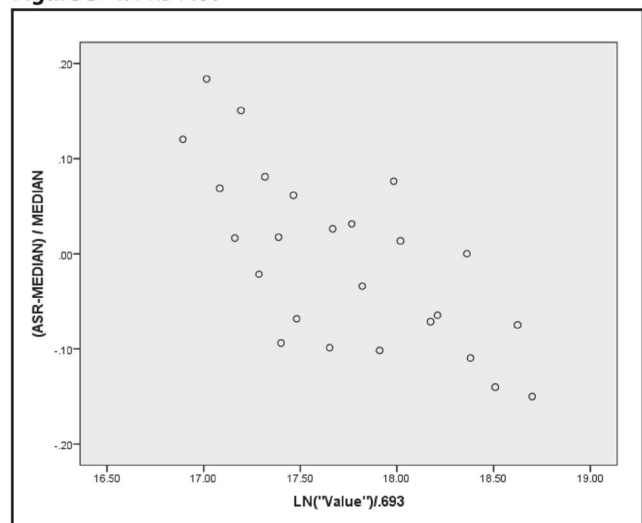
Table D-2. Ratio data with outlier

SALE	PRICE	ASMT	ASR
1	50,000	45,000	0.900
2	100,000	100,000	1.000
3	150,000	165,000	1.100
4	200,000	180,000	0.900
5	250,000	250,000	1.000
6	300,000	330,000	1.100
7	350,000	315,000	0.900
8	400,000	400,000	1.000
9	450,000	495,000	1.100
10	500,000	450,000	0.900
11	550,000	550,000	1.000
12	600,000	660,000	1.100
13	650,000	585,000	0.900
14	700,000	700,000	1.000
15	750,000	825,000	1.100
16	2,500,000	1,875,000	0.750

Table D-3. Ratio statistics with and without outlier

Ratio Statistics for 15 Sales (No Outliers)							
Ratio Statistics for ASMT / PRICE							
Sales	Mean	Median	Weighted Mean	Minimum	Maximum	PRD	COD
15	1.000	1.000	1.008	.900	1.100	.992	.067

Model	Coefficients		t	Sig.	95.0% Confidence Interval for B	
	B	Std. Error			Lower Bound	Upper Bound
1 PRB	.020	.020	1.032	.321	−.022	.063

Ratio Statistics for 16 Sales (1 Outlier)							
Ratio Statistics for ASMT / PRICE							
Sales	Mean	Median	Weighted Mean	Minimum	Maximum	PRD	COD
16	.984	1.000	.932	.750	1.100	1.056	.078

Model	Coefficients		t	Sig.	95.0% Confidence Interval for B	
	B	Std. Error			Lower Bound	Upper Bound
1 PRB	−.011	.021	−.520	.611	−.056	.034

Appendix E. Sales Chasing Detection Techniques

As long as sold and unsold parcels are appraised in the same manner and the data describing them are coded consistently, statistics calculated in a sales ratio study can be used to infer appraisal performance for unsold parcels. However, if parcels that sell are selectively reappraised or recoded based on their sale prices or some other criterion (such as listing price) and if such parcels are in the ratio study, sales ratio study uniformity inferences will not be accurate (appraisals will appear more uniform than they are). In this situation, measures of appraisal level also will be unsupportable unless similar unsold parcels were appraised by a model that produces the same overall percentage of market value (appraisal level) as on the parcels that sold based on consistently coded descriptive and locational data.

Assessors and oversight agencies do not need to employ all the detection techniques described in this appendix, but should consider implementing at least one procedure. In some cases, access to assessment information for all properties is necessary to perform the suggested techniques. Agencies that do not have access to these data are at a disadvantage, but should still implement detection techniques, such as those described in sections E.3 and E.4, which do not require such comprehensive assessment information.

E.1 Comparison of Average Value Changes

If sold and unsold properties within a specified group are appraised in the same way, their appraised values should reflect similar average percentage changes from year to year. Accordingly, changes in appraised values for sold and unsold parcels can be compared to determine whether sold parcels have been selectively appraised. Alternatively, the average percent change in value for sample parcels can be compared to that for the population of properties within a specified group or stratum for an indication of selective reappraisal.

For example, if sold parcels are considered representative of a stratum and appraised values increased an average of 10 percent while appraised values for unsold parcels in the same stratum increased an average of only 2 percent, "sales chasing" is a likely conclusion. At a more sophisticated level, the distribution of value changes for sold and unsold parcels can be compared, or statistical tests can be used to determine whether the distributions are different at a given level of confidence.

Statistical significance in the absence of practical significance may be moot. In large samples, small differences in the magnitude of assessed value changes on sold and unsold parcels can be proven to be statistically significant, yet the actual differences may be slight. Therefore, it is prudent to establish some reasonable tolerance, such as 3 percentage points (e.g., a change of 6 percent for sold properties and 3 percent for unsold properties), before concluding that a meaningful problem exists. Such tolerance applies to other detection techniques discussed below.

E.2 Comparison of Average Unit Values

If sold and unsold parcels are appraised equally, average unit values (for example, value per square foot) should be similar. An appropriate test (Mann Whitney or t-test) can be conducted to determine whether differences are significant.

E.3 Split Sample Technique

In this technique, two ratio studies are performed, one using sales that occurred before the appraisal date and one using sales after the appraisal date, both adjusted for date of sale as appropriate. Except for random sampling error and any error in time adjustments, results of the two studies should be similar. Sales chasing is indicated if the results of the first study are consistently better than those from the second. In such a case, the second study is still valid; the first study should be rejected.

E.4 Comparison of Observed versus Expected Distribution of Ratios

Assuming the ratio studies are based on sales that have been properly adjusted for time and other factors, a strong indication of the likelihood of "sales chasing" can be obtained by computing the proportion of ratios that would be expected to fall within a particular narrow range of the mean given the lowest likely standard deviation (although this depends somewhat on the assumption of a normal distribution). For example, with a standard deviation of 5 percent given a normal distribution, about 32 percent of the ratios would be expected to fall within ±2 percent of the mean (for example, between 98 and 102 percent, given a mean of 100 percent). Except in highly constrained or well-behaved real estate markets, many appraisers consider such a low standard deviation, corresponding approximately to a COD of 4 percent, to be unachievable. Regardless of the distribution of the ratios, the likelihood is extremely low that there would be a sufficiently representative sample with more than this proportion of ratios in such a narrow range. If such is the case, "sales chasing" is a likely conclusion. Sometimes other processes through

which adjustments to assessments on selling parcels are more pronounced than on the population as a whole mimic the effect of sales chasing, such as more intensive reviews of sales than non-sales. Regardless of the practice, the representativeness of the ratio study is called into question and additional tests should be instituted.

Although samples may not be normally distributed, in which case equivalently precise proportions of expected ratios around the median cannot be determined, the 32 percent concentration is very conservative. Finding such a high concentration of ratios around any measure of central tendency is a strong indicator of sales chasing or of a non-representative ratio study. In addition, when the distribution of ratios is bimodal or multimodal, similar significant concentrations of ratios around these modes can indicate selective reappraisal or sales chasing.

Table E-1 demonstrates the conservative nature of the 32 percent concentration. If the minimum achievable COD is, in fact, higher than 4 percent for the strata or property class being analyzed, then even lower concentrations could indicate sales chasing, and previously discussed investigative procedures should be instituted. One disadvantage to this procedure is that it can be misleading when applied to small samples. Therefore the method should not be employed for sample sizes less than 30.

Even when critical proportions of ratios shown in table E-1 are exceeded, further investigation should be conducted before concluding that sales chasing has occurred.

E.5 Mass Appraisal Techniques

Provided sales are sufficient in number, oversight agencies can develop mass appraisal models to apply to a random sample of unsold properties or to the population of properties that are represented by the sold properties. An independent multiple regression or other automated calibration techniques can be used to develop the models. An appraisal ratio study is then conducted for the unsold parcels by using values predicted by the independent models as indicators of market values. This approach has the following advantages:

- It is objective and rooted in the market.

- The models can be reviewed for sufficient reliability before being applied to the unsold parcels.

- The technique yields measures of central tendency, which can be compared against those produced by the sales ratio study and tested for compliance with standards for the level of appraisal.

- The technique takes the form of an appraisal ratio study but avoids the time and expense of single-property appraisals.

Reliability of this method depends on the accuracy and independence of the mass appraisal models used to generate the value estimates. The models must be consistent with appraisal theory and reviewed for sufficient reliability by examining goodness-of-fit statistics. The models should be independent of those used for assessment purposes.

Table E-1. Example of critical ratio concentrations indicative of sales chasing or similar practices

Minimum achievable COD	Standard deviation assuming normal distribution and mean ratio of 100%	Critical proportion of ratios*	z score based on ± 2% range (Absolute value)	Expected proportion of ratios below 0.98	Expected proportion of ratios below 1.02	Expected proportion between 0.98 and 1.02 (within ± 2% of central tendency)
1.6%	2.00%	69	1.0000	0.1587	0.8413	0.6826
4.0%	5.00%	32	0.4000	0.3446	0.6554	0.3108
5.0%	6.25%	26	0.3200	0.3745	0.6255	0.2510
6.0%	7.50%	22	0.2667	0.3949	0.6051	0.2102
7.0%	8.75%	19	0.2286	0.4110	0.5896	0.1801
8.0%	10.00%	16	0.2000	0.4207	0.5793	0.1586
10.0%	12.50%	13	0.1600	0.4364	0.5636	0.1272
12.0%	15.00%	11	0.1333	0.4467	0.5530	0.1063
14.0%	17.50%	10	0.1143	0.4545	0.5455	0.0910
16.0%	20.00%	8	0.1000	0.4602	0.5398	0.0796

Given the assumption that the COD shown represents the minimum achievable COD for the property type, class, or strata being analyzed with the ratio study, sales chasing (or a similar distortive procedure) is very likely if the concentration of ratios with ± 2% of a measure of central tendency, such as the median or a mode, or 100%, equals or exceeds this value. This proportion is based on values of the standard normal distribution function and assumption that sample size is greater than 30. The critical number equals the integer immediately exceeding the expected proportion.

Appendix F. Alternative Uses for Ratio Study Statistics

In addition to the use of statistical measures to determine underlying assessment level and uniformity, comparisons between measures can provide useful information about sample representativeness, the distribution of the ratios, and the influence of outliers. For example, by comparing the mean and weighted mean, even without determining the PRD, the analyst should be aware that a large difference between these two measures indicates probable influence of atypical ratios on high-priced properties. This in turn could mean that outliers are still present in the sample and that the sample is not representative. Alternatively, it could indicate systematic appraisal error in the appraisal of properties within a particular price range. The geometric mean-to-mean relationship can provide similar information, especially about the presence of very low ratios, which have a greater influence on the geometric mean. The relationship between the COD and COV can provide similar additional guidance. This standard chooses the COD as the primary recommended measure of uniformity. This choice reflects the expectation of non-normal distributions of ratios. Despite this consideration, it is useful to recognize that, in a normal distribution, the COV is approximately 1.25 times the COD. When the COV/COD ratio exceeds 1.25, the likely cause is a small number of very high ratios, which may again be non-representative.

It is incumbent on the analyst to review the ratio study sample to attempt to provide a representative sample. Comparisons of statistics, such as those given in this appendix, provide an additional tool to help the analyst in this regard.

Appendix G. Legal Aspects of Ratio Studies

Property taxation is governed by federal, state, and provincial constitutions, statutes, and administrative rules or regulations, many of which require uniform treatment of property taxpayers. Ratio studies play an important role in judging whether uniformity requirements are met. Relevant Canadian Federal statutes based on the Constitution Acts of 1867–1975 provide that municipal councils cannot discriminate between taxpayers of the same class within municipalities.

Relevant United States federal provisions include the Bill of Rights, the commerce clause of the United States Constitution, the Fourteenth Amendment, and the Tax Injunction Act (28 U.S.C. § 1341). Together they guarantee basic protections and due process while still granting states the authority to classify property and grant reasonable exemptions. Many constitutions have clauses that require uniformity in the assessment and taxation of property, although some jurisdictions, either by constitution or statute, permit certain differences between classes. Ratio studies provide a gauge of whether uniformity requirements are being met.

A key U.S. federal statute relating to ratio studies is the U.S. Railroad Revitalization and Regulatory Reform Act ("4-R Act") of 1976 (49 U.S.C. § 11501). The 4-R Act requires that rail transportation property be assessed for tax purposes at no more than 105 percent of the assessment level of other commercial and industrial property in the same taxing jurisdiction. Similar federal statutes relate to air transportation property, motor carriers, and bus lines (49 U.S.C. §§14502 and 40116).

The 4-R Act provides that ratio studies be used to measure alleged discrimination. In such cases, as in any ratio study, the purpose of the study must be clearly defined and the study must be conducted so that it accurately evaluates the issues at hand. Important issues in ratio studies conducted pursuant to the 4-R Act include the proper definition of "other" commercial and industrial property, screening and adjustments to sales data, proper measures of the level of appraisal, and the combining and weighting of centrally valued and locally assessed properties.